Articles and Scheduling for Student Seminar in Combinatorics: Linear Complementarity

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1 Seminar Schedule

The schedule is meant to be tentative and may be modified depending on the progress of our seminar. Students may use blackboards, overhead projector, or beamer for presentation. The lecture room is HG E 33.3 (Tuesday 10-12).

Date	Article	Presenter(s)
September 15	overview, initial planning	Komei Fukuda
September 22	fixing teams and planning	Komei Fukuda
September 29	Preparation (no seminar)	
October 6	QP duality [5]	team 1 (Bausch and Heins)
October 13	LCP classics [6, pages 103-114]	team 2 (Buchmann and Mishra)
October 20	LCP classics [6, pages 114-124]	team 3 (Akeret)
October 27	NP-completeness [4] [20, Section 3.4]	team 4 (Dummermuth)
November 3	PSD- and P-matrices [7, Sec 3.1–3.3]	team 5 (Göggel and Schneebeli)
November 10	SU-matrices [7, Sec 3.4–3.6]	team 6 (Allemann)
November 17	P(& SU)-LCP may not be hard [22, 16]	team 7 (Schiesser)
November 24	Randomized algorithms [15, 23]	team 8 (Sidler and Towa)
December 1	Two polynomial cases [11]	team 9
December 8	Oriented matroids and LCP [17]	team 10 (Leroy and Stenz)
December 15	Combinatorial char. of K-matrices [12]	team 11 (Gleinig)

2 Presentations and Teams

In this seminar, we study some of the most critical literatures on linear complementarity with a strong emphasis on combinatorial and algorithmic aspects. Our focus on the complexity of subclasses of the linear complementarity problem (abbreviated by LCP) is closely related to understanding the complexity of linear programming and convex quadratic programming. The key question is whether there exists a strongly polynomial algorithms for these fundamental problems. Below we list the papers and book sections that we will study together. We may add some new articles for presentation, such as those listed in the references, if we are expected to profit from them.

We will have 11 teams, each of which consists of one or two students. A team of two can be made with mutual consensus, but we will have alternative way to make a team by random selection. Each presenter gives a talk on the assigned material for 45 to 90 minutes. Please have a pause of at least 5 minutes in the middle.

Linear and convex quadratic programming Two of the most important special cases of LCP is linear programming (LP) and convex quadratic programming (convex QP). The paper [5] addresses the duality of convex QP in symmetric form. This duality generalizes the well known duality of LP.

Classical literatures on LCP Two epoch making papers on LCP are one by Lemke [21] and the other by Cottle and Dantzig [6]. They both witness the excitement and beauty of the LCP framework that unify many fundamental problems in optimization and game theory.

One caution is in order. Sometimes, old literatures **may not be careful about matrix or vector multiplications**. In particular, [6] writes zw for the proper $z^T w$ or $z'w$ that represent the inner product of two vectors z and w. Similarly, it uses xDx for the correct $x^T Dx$ which represents the quadratic form of a square matrix D.

In order to understand the concrete examples in [6], one must know how to compute pivot operations by hand or by computer. The lecture notes [14, Section 4.3] is a good reference for pivot operations for linear systems.

The LCP book There is only one book on LCP that covers extensive results on the subject. It is [7] by Cottle, Pang and Stone. It is an excellent literature and everyone in the seminar should keep an electronic copy as a general reference. Yet, our main interest lies in the combinatorial aspect of LCPs and this book itself is not sufficient for the purpose. We use only a few sections in our seminar, namely, Sections 3.1–3.6 concerning the important classes of square matrices such as PSD- (positive semidefinite), P- and SU- (sufficient) matrices.

NP-Completeness LCP The general LCP is NP-complete by [4]. A stronger result on the strong NP-completeness is given in [20]. Both uses a polynomial reduction of a well-known NP-complete combinatorial optimization problem to an LCP.

Subclasses of LCP There are many different classes of LCP that have been studied extensively. The book [6] addresses these classes and their relationships. Some classes such as P-matrices, K-matrices, PSD-matrices have been subjects of the classical linear algebra and optimization. The class of sufficient (SU-) matrices contain both the classes of P- and PSD-matrices plays a significant role in understanding the well-behaving LCPs.

P-LCP cannot be hard Megiddo [22] remarked that the P-LCP cannot be hard unless NP=coNP. On the same token, one can argue that the SU-LCP cannot he hard unless this very unlikely scenario is true. This way of looking at LCP leads to the EP-theorem on SU-LCP studied in [16].

Combinatorial Abstraction of P-matrix LCP There is a graph theoretical way to study the LCP with P-matrices, known as the unique sink orientations of hypercubes. This framework [26, 23, 11, 12] looks at the property of orientations of hypercube graphs induced by P-matrix LCPs.

Analysis of Randomized Algorithms There are some positive and negative results on randomized algorithms for LCPs. Fukuda and Namiki [15] shows the randomized Murty's algorithm terminates in an expected polynomial time for a well-known exponential case. On the other hand, Morris [23] showed an exponential expected behaviour of some randomized algorithm for P-matrix LCPs.

Combinatorial Abstraction of LCP The notion of oriented matroids (OM) provides an ideal combinatorial abstraction (OMCP) of LP and LCP [27, 28, 17]. In fact, it might be possible that if there is a combinatorial polynomial algorithm for SU-LCP, it might be polynomial even for the larger class of SU-OMCP. To achieve this is an ultimate goal for any ambitious researcher in optimization that should have strong consequences not only to the theory but to the practice.

3 Final Report

Each student (not a team) must submit a final report in pdf of 5 to 10 pages written in latex covering the presented material, detailed proofs and possibly your conjectures, within four weeks after the presentation. An exception is granted for a 2-person team to write a joint report, provided that the members proved that the members work closely together and there is no reasonable way to write separate reports.

4 Articles Online

In addition to making use of web search engines, each student is expected to learn to use the AMS (American Mathematical Society) **MathSciNet** database to search for articles you wish to read: http://www.ams.org/mathscinet/index.html . Even if the database item of the article has no link to the pdf version, please do not give up. Go to the journal site and search there for the pdf. By now, most of the important articles are available online. Please make sure that your network connection is established within the ETH domain (by possibly using VPN from home).

5 Office Hours

The default office hours are Friday 10:00–12:00 and 15:00–16:30. Please send your reservation request by e-mail. Other day/time might be available but you have to make an appointment at latest one day before.

Fukuda's office is CAB G 33.3, which is near the entrance door furthest away from the main building HG.

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