What Mathematics Should "Everyone" Know and Be Able to Do?

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This essay presents ideas for teaching what some people are now calling "quantitative literacy." Some of the ideas are old hat—project-based collaborative learning, teaching in context, and using computers. Some are radical—banning the use of *x* and *y* as variable names until after college algebra. An early step is to determine whether a "canon of empirical mathematics problems" can be defined, doing for quantitative literacy what the canon of literature does for cultural literacy. Some will argue that this quest departs too radically from current mathematics education to be feasible, but the current algebra curriculum has its own canon. An amusing book, *Humble Pi,* sets it out: age problems, canoe problems, planes meeting in mid-continent, and so on.[1] Why not a comparable set for quantitative literacy?

As befits an essay on empirical mathematics (a term I prefer to quantitative literacy), a table of specific types of examples is provided. It also is appropriate to cite some data. Over the last few years, inner-city Baltimore students were taught quantitative literacy in their algebra courses. They outperformed traditionally taught students by a wide margin. They took and passed Algebra II at a greater rate, received higher grades, were absent less, and were more likely to graduate and to go on to college.[2]

This essay concludes that it is better to teach mathematics inductively. Let students first learn the power of mathematics in specific examples. Later, they can appreciate mathematics' power to generalize. The inductive approach is more likely to succeed than the current deductive process, in which general rules are taught first and applications—selected from the canon noted in *Humble Pi*—are of secondary importance.

The Challenge

The first order of business is to demonstrate that mathematics education is inadequate to today's challenge. The challenge exists because of mathematics' growing importance for both economic and citizenship reasons. It is no accident that, along with reading, mathematics is one of the two subjects that always are required on standardized tests. This implies that it is important for everyone, not only for those few who "love" the subject and grow to see the beauty in it.

Two hundred years ago, only merchants, engineers, surveyors, and a few scientists were mathematically literate. Merchants had to calculate the price of cloth "2 yards 1 foot 4 inches square at 3 pence 2 farthings the square foot."[3] Military engineers had to determine the angle needed for a cannon to project a missile over a moat. Surveyors had to lay out site lines. Isaac Newton needed to invent calculus to solve his physics problems. How many Americans are now mathematically literate is an arguable question. By some estimates, it is less than one-fourth: that is how many adults achieve levels of 4 and 5 in the National Adult Literacy Survey (NALS) and International Adult Literacy

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Survey (IALS) studies,[4] levels at which they can perform two or more sequential operations or tasks of similar complexity.

But the rigors of international competition have changed policymakers' views on the issue. Ten years ago, the first President Bush and the governors (including then-Governor Bill Clinton) met in Charlottesville, North Carolina to set education goals. By the year 2000, U.S. students were to be first in the world in science and mathematics. But now we are well past 2000, and by any measure we have not met that goal. National Assessment of Educational Progress (NAEP) scores in mathematics (for 17-year-olds) in 1996 were only 3 percent higher than in 1982. The average NAEP score was 307, meaning that the average 17-year-old can compute with decimals, use fractions and percentages, recognize geometric figures, solve simple equations, and use moderately complex reasoning. The averages among blacks (286) and Hispanics (292) were below 300, meaning the ability to do no more than perform the four arithmetic operations with whole numbers and solve one-step problems.[5] Over half of the students entering the California State University system need to take a "developmental" mathematics course. Over one in four college freshmen feels a need for tutoring or remedial work in mathematics. This compares to one in 10 for English, science, and foreign language.[6]

Teaching is part of the problem. International comparisons of mathematics teaching find U.S. methods to be inferior to those used by Japanese or German teachers. The nation faces a severe shortage of primary and secondary school mathematics teachers and an overabundance of those trained to be college mathematics teachers. In 1998, 18 percent of high school mathematics teachers did not have a major or minor in the subject.[7] At the same time, only 2 percent of college freshmen expect to major in the physical sciences. This 2 percent includes only 0.5 percent in mathematics.[8] Mathematics teachers are disconnected from other faculty in many schools and colleges.[9] As a result, mathematics lacks context and other courses lack mathematics.

Although they may not know the reasons why, generations of American students have been convinced that something was amiss with mathematics classes. Many a parent has heard their teenage children complain, "I hate math; it's boring and hard. Why do I have to learn math, it's so useless. . . ." Many parents are sympathetic. They themselves finished their last required mathematics course in high school or college with expressions of relief, not commitments to take another mathematics course as an elective.[10]

These parents often are mathematically inadequate at their own jobs and in other aspects of their lives. They do not understand statistical quality processes, cannot follow political candidates who speak of "weighted averages,"[11] and cannot make sense of alternative strategies for financing their own retirement. They would express wonderment if by some small chance they ever met a mathematician who spoke of mathematical beauty (although seeing *A Beautiful Mind* might have an impact). Our society pays a high cost for the general lack of mathematical competence.

What is wrong? The way middle school teachers teach fractions provides a clue. They teach their students to add fractions by:

First finding the lowest common denominator.

Then converting all fractions to that denominator.

Then adding the numerators.

Finally, reducing the answer, if possible.

Nobody does that outside the schoolroom. Imagine a school cafeteria in which the selected items totaled three quarters and three dollars and four dimes. The schoolroom method would be to change all these in for nickels. Or go to the shop. Maybe the problem is adding one foot and 8 and 1/16 inches to 6 and 1/4 inches. Would any carpenter change it all into sixteenths? It is a very rare situation when anyone needs this method, say to add odd fractions such as fifths and sevenths.

Mathematics teachers might say they want a "general solution" so that students could add twentieths and sixteenths. A thoughtful student might respond, "Yeah . . . like when?" We are generally using the decimal system (for money or where the metric system prevails) or the English system of measurement. The practical result is *not* a universe of students who can solve a universe of fraction problems. Instead, it is a great many students who learn (about the sixth grade) that they "can't do math" and demonstrate that truth by being unable to solve either the cafeteria or shop problems.[12]

Nor is the general abstract approach to mathematics necessarily a big favor for those who "love" mathematics or science. "No scientist thinks in equations," Albert Einstein said. Einstein employed visual images and muscular feelings. The mathematician S. M. Ulam said that he uses "mental images and tactile sensations to perform calculations, replacing numerical values with the weights and sizes of imagined objects." Joshua Lederberg becomes "an actor in a biological process, to know how [to] behave as if I were a chromosome."[13]

Evidence from mathematics assessments is consistent with the theories (and data) from cognitive science: it is better to build abstract thinking on a concrete base. Adding ethereal fractions or solving $3x = 9$ by eliminating references to concrete objects or phenomena removes the connection to nonsymbolic ways of thinking about mathematics. For too many, the bloodless abstraction makes it impossible to learn the subject. Others can remem-

ber, via rote recall and long enough to pass the final, how to plug in numbers and chug through the formula.[14] Do not, however, ask either the passing or failing students to apply the technique to a real-life problem. Weeks after the final, they cannot even remember that the formula exists. They have no way to recall the formula from long-term memory. Imagine asking most adults the formula for solutions to the quadratic equation or, worse yet, what real-life process is described by the equation. Nor does the problem disappear as students take higher-level courses: ". . . undergraduates never learn how calculus relates to other disciplines, much less the real world."[15] The situation is untenable for two kinds of students: those who do not like mathematics and those who do.

Thomas Berger of Colby College, former chair of the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America, speaks of the "mathematization of society" as symbolic models become the basic tools of engineers, medical researchers, and business executives.[16] These models, however, simulate real-world processes and systems. They are used to allocate resources, design technology, and improve system performance. Mathematization increases the significance of higher-level skills for most citizens. Successful workers, citizens, and consumers need to know how to solve problems, analyze data, and make written and oral presentations of quantitative results.

How does the traditional pre-calculus curriculum serve these purposes? How valuable, actually, is calculus? Even mathematicians often replace calculus with finite mathematics to take advantage of computer technology.[17] The same technology can handle both the traditional tasks of manipulating formulas and performing long computations. Spending too much time teaching humans to solve problems better handled by machines is not a wise strategy when there is so much to know.

Conceptualizing how a problem can be stated mathematically has become (and, indeed, always was) more valuable than factoring a polynomial or taking a derivative. The tasks most college graduates face demand quantitative literacy (a.k.a. empirical mathematics)—a way with numbers and comfort replacing concrete realities with symbols—without forgetting the reality beyond the symbols. In other words, banish x and y from mathematics class at least until the completion of college algebra. Use, instead, letters (even Greek letters) that stand for something students can understand or picture: v for velocity, d for distance, P for price, n for number, p for profit, and so on.

This idea is not so radical. In his introduction to *Why Numbers Count,* Lynn Arthur Steen referred to scientific mathematics in which mathematical variables always stand for physical quantities—"a measurement with a unit and implicit degree of accuracy."[18] In the same volume, F. James Rutherford said, " . . . citizens need to possess certain basic mathematical capabilities *understood in association with* relevant scientific and technological knowledge."[19]

Mathematicians, of course, want their students to understand mathematics' power to solve general problems, ones that are not rooted in a specific problem. That point can be made and demonstrated near the end of the mathematics course. Teach that the equation for velocity can be used in many contexts relating to change. True generality can be saved for those mathematics students who still will be taking mathematics courses in their junior year in college. (Indeed, there might be more such students if mathematics were less abstract in the earlier years of school.) *Make each year of mathematics instruction worthwhile in itself, not just preparation for the next mathematics course.*

Many critics of this point of view believe it does not credit the power of abstract mathematics to be generally applicable. But—as guns don't kill but people with guns do—mathematics does not apply itself: mathematically competent individuals do. Individuals require enough competence and creativity to structure a problem mathematically and to know when and how to use the tools. On that basis, the current approach to achieving widespread mathematical competence is failing. The real issue is whether mathematics should be taught inductively—from the concrete to the abstract—or the other way around, as it often is today. The answer can be found empirically, not theoretically. Which approach will meet education's goals of productive workers, engaged citizens, and well-rounded individuals who continue to learn after graduation?

Quantitative Literacy: Goals and Objectives

What mathematics should "everyone" know and be able to do? The National Council of Teachers of Mathematics (NCTM) has been attempting to answer this question for more than a decade as council members developed mathematics standards. Wisely, NCTM included the ideas of real-world problem solving and being able to communicate in the language of mathematics. Its report, however, is built around standard mathematical topics, algebra, geometry, calculus, statistics, and so on. The problems were what we might expect: Solve $x^2 - 2x - 7 = 0$, or derive the equation describing the motion of a Ferris wheel car. Few individuals work on either of these problems outside of a school situation. One helpful criterion is to restrict problems to those that American workers get paid to solve, those that American citizens should have informed opinions about, or those that American consumers actually need to solve.

Consider the challenge posed by Berger's mathematization of society. What if Berger's problem were stated in the language of mathematical optimization? (Surely, mathematicians should applaud using their discipline to analyze this problem.) In the workplace, industrial psychologists analyze the frequency and criticality (sometimes called importance) of tasks and the various skills needed to carry them out. The human resources department then either hires folk with the more important skills or trains staff in those skills. Transfer this thinking to the school situation in which the challenge is how best to prepare students for their life beyond the schoolhouse walls. Think first of the student's professional life. The optimizing school would seek to maximize the benefit that is a function of:

1. The frequency of having any particular job or career;

2. The probability that any particular problem will arise in that job;

3. The criticality of having the appropriate mathematics skills to solve the problem; and

4. The economic importance of solving the problem properly.

The mathematics curricula should seek to maximize the benefits under the constraints of time, talent, and materials.[20] The trick is figuring out the problems that students exiting formal education after grade 14 have a significant likelihood of having to solve in their careers. In an earlier paper, I estimated that advanced mathematical content was required in about 4 percent of jobs; that is the percentage of engineers, scientists, computer analysts, financial analysts, and accountants in the labor force.[21] I also suggested that the weighted probability of a problem arising should be 1 percent or more before it should be put in the canon of problems that all students should be able to solve. Applying this criterion to curricula would mean that 1.5 million persons would have to encounter a problem in their lifetimes before it was worth including in the canon.

As noted before (and repeated for emphasis), careers and work are not the only economic reasons for taking mathematics. In a consumer society, successful individuals make wise consumption, saving, and investment decisions. The equation needs to be supplemented, therefore, with the probability of making decisions in these domains, the frequency with which the decisions will be made over a lifetime, the criticality of mathematics for making the decisions wisely, and the economic cost of the decisions.

Education also has responsibility for preparing students for their roles in noneconomic domains. A democracy needs citizens who participate with awareness and understanding; surely this is a mandatory objective of publicly funded education. Citizens should be able to understand the *New York Times,* the president's State of the Union speech, and the school district's budget. Comprehending statistics and reading charts and tables are essential. There are also personal reasons for knowing some mathematics. Now we have to add the importance of encountering problems in these two noneconomic domains to the benefit equation. Instead of only economic costs, political and social costs need to be considered.

The challenge is to identify important, frequently encountered problems that cannot be efficiently solved without using mathematics. In other words, the challenge is to identify a canon of mathematical problems analogous to the canon in literature. David Denby, film editor for the *New York Times,* wrote a controversial book about his midlife experience of returning to Columbia University to revisit the literature canon of the Western world. He read literature from Homer and the Bible to Virginia Woolf.[22] Can anything similar to the literary canon be described for mathematics? What problems have been, and will be, relevant for centuries and across cultures?[23]

Well, some problems have been around for a while. We can imagine that the Egyptian pharaohs had problems of budgeting and scheduling construction of the pyramids. The biblical story relates Abraham's negotiating the price for the cave of Machpelah as a burial place for him and Sarah. Penelope's suitors would have benefited from knowing something about the rate of change in the area of the cloth she was weaving. Similar problems will be around for the next new millennium if humans last that long.

The classes of problems also should remain fairly constant across cultures and up and down the organizational ladder. Problems of budgeting apply from the fourth grade to the CEO (and, of course, to the president and Congress). The problem only becomes more complex as variables multiply and uncertainty enters the equations. Of course, the way budget and schedule problems are solved changes—King Tut did not have a spreadsheet or Harvard Project Planner. With computers, mathematicians may choose linear programming rather than calculus to solve numerical optimization problems.

A Scheme

E. D. Hirsch is widely known for his educational canon. He lists things students should know at each grade level. I will try to avoid such laundry lists and suggest, instead, a structure in which types of problems can be placed or developed. This structure can accommodate a range of difficulties for each of the problem types. Some examples in the range are suitable for different school grades. A second grader may, for example, be taught something about schedules while mathematics post-docs may struggle with variations of the traveling

salesman problem. The particular examples we put forth are suitable for grades 11 to 14 in school and for college.

The structure builds on the SCANS taxonomy, a set of competencies developed in 1991 by the Secretary's Commission on Achieving Necessary Skills (SCANS).[24] These competencies—a better term is problem domains—are quite broad and were intended to accommodate a full range of situations from entry-level to CEO. The five SCANS domains and the subdomains that require quantitative literacy are:

- *Planning problems.* Allocating money (budgeting), time (scheduling), space, and staff.
- *Systems and processes problems.* Understanding, monitoring, and designing social, physical, or business systems.
- *Interpersonal problems.* Working in teams, negotiating, teaching, and learning.
- *Information problems.* Gathering and organizing data, evaluating data, and communicating, both in written and oral form.
- *Technology problems.* Using, choosing, and maintaining equipment of any type.

Solving problems in any domain requires a foundation of basic (reading, writing, listening, speaking) and higher-order (problem-solving, decision-making, creativity, and mathematical) skills and personal qualities such as integrity, sociability, and self-management. Performing in a real situation often demands solving problems in more than a single domain.

The SCANS categorization must be approached with care and common sense. It is an abstract structure imposed on an endlessly complex reality to help organize examples. The categories are only reasonably comprehensive and relatively disjointed. In the table at the end of this essay, many problems could easily fall into different box from the one into which they were put.

Other structures can be, and have been, devised. The U.S. Department of Labor developed O*Net, the New Standards group developed Applied Academics, and a number of states and occupational groups developed their own standards. The manufacturing and sales and service sectors developed standards under the guidance of the National Skills Standards Board. Equip for the Future (EFF), a project of the National Institute for Literacy, has extended similar standards to define the literacy needs for work, citizenship, and parenting. Basically, all these standards are variations on the same theme and crosswalks have been developed to link them. All the good ones strike a balance between an endless unstructured list of examples and an almost equally endless list of narrow categories. The best structures acknowledge that human memories are generally limited to recalling a list of five to seven items.

I chose the SCANS taxonomy for the following four reasons:

1. It is the best-known and most widely used structure available.[25]
2. It is one of the only structures that have widespread recognition across the academic and occupational standards that have been developed in the United States and other countries.
3. It is easily modified to suit standards developed under other structures, such as state standards for mathematics.
4. It is easily extended to roles other than career and work.[26]

Four "roles" are used to fill out the structure. For most students and policymakers, the role of work and careers is of foremost concern. If not for this role, mathematics departments would be competing for students with literature and art departments, not with computer sciences. Unlike most school-based problems, real work-based problems usually cannot be solved in a few minutes but take hours of sustained effort. Preparation for this role requires that students engage in long-term projects. The ability to carry out such tasks has been noted by a recent National Academy of Sciences effort to define computer literacy and fluency.[27] Academy member Phillip Griffiths, director of the Institute for Advanced Studies in Princeton, speaks to the need for the mathematically competent to function in various SCANS domains. "We asked . . . [about] . . . science and engineering PhD's. The employers told us that . . . they find shortcomings: . . . communications skills . . . appreciation for applied problems . . . and teamwork. . . . Skills like project management, leadership . . . interpersonal skills . . . computer knowledge." Students will have to work in teams, use computers to solve problems, and make oral and written presentations if Griffith's requirements are to be met.

Many mathematics teachers may decry my emphasis on work and careers, so I want to acknowledge, once again, the importance of other domains (without relinquishing the idea that the primary force behind the nation's emphasis on mathematics is economic). Individuals also require mathematics to succeed in their second important role as consumer. Some buying problems can and must be solved quickly and on the spot. Comparing the price per ounce for similar products sold in differently sized bottles, understanding discounts, approximating a large restaurant or hotel bill are examples. Other problems, such as comparing retirement or health plans or comparing mortgage rates may take more time (although the Internet can speed things up).

Citizenship, the third role, is a noneconomic role, one that is mandatory if public funds are to be used for education. Informed discussions about the adequacy of Social Security, the growth rates of spending and taxing, health policy, science policy, and a host of other topics are impossible for the mathematically illiterate.

Finally, mathematics can provide personal pleasure. This is the personal role. Pleasure can take the form of appreciating pure mathematics itself, reading about a variety of subjects that require mathematical or computer literacy for full understanding, or employing mathematics for leisure activities from astronomy to sailing to carpentry.[28] In this domain, mathematics educators will have to compete with music, arts, and physical education.

Thoughts on Breadth Versus Depth and Pedagogy

Some believe that students, after learning all the mathematical topics in the current curriculum, can put them together for applications such as those contained in the Appendix. That belief is not based on empirical data. Students—and, unfortunately, many mathematics teachers—simply cannot do it. People learn how to put together budgets by making a budget. Projects are necessary. This raises the hoary problem of coverage. It brings up the optimization problem stated earlier: how to provide the most value to the student and society under constraints of time and resources.

Massachusetts Institute of Technology mathematics professor Arthur Mattuck pointed out that teaching through modeling is difficult. Models must be sophisticated to capture interest. The more time spent building and understanding the model the less time for coverage. "In the end," he said, "very little mathematics gets done."[29] One response to Mattuck is this: "What if covering Taylor's expansion rather than a more interesting modeling project means that fewer students will take the next mathematics course?"[30] This clearly illustrates the trade-off.

Brain research has shown again and again that retention of information requires context. Unless a student can provide a mental map, developed by making connections, memory of facts or skills will soon dissipate. This holds true whether people are trying to remember names, formulas, or how to solve a differential equation.

The dilemma of coverage was highlighted in the Third International Mathematics and Science Study (TIMSS) international comparison of mathematics teaching and learning. American textbooks are thick, Japanese thin. We cover more topics lightly and they cover fewer topics in depth. Japanese students surpass those of most other countries in international mathematics assessments. We are in about the middle of the pack.[31]

The lessons of TIMSS apply as well to empirical mathematics. Learning how to solve the problems shown in the Appendix will take more time and is unlikely to be done well unless certain other topics are eliminated or at least postponed until higher college levels when career choices are clearer. Recall, yet again, that we are trying to maximize usefulness under constraints of time, money, and talent.

Even if the curriculum is revised to fit, pedagogy will have to change. Managing groups of students working collaboratively on a project is different from lecturing. Computers and the Internet will probably be part of the instructional materials. Teachers who offer these sorts of classes find it to be more work but more rewarding than using traditional methods. Creating projects is time consuming. Most mathematics teachers do not have the time or inclination (or talent) to create realistic projects. Electronic and paper libraries (such as the Harvard Case Studies) have and can be created.[32]

Coverage of mathematics topics will also have to be reduced if the culture of mathematics is to fit into the limited time that can be devoted to the subject. If all students are going to understand mathematicians' discovery processes,[33] there will be less time to understand the discoveries themselves. Consider, for example, the fifth-grade lesson $C=\pi D$. It is much less interesting than $C/D = \pi$. With the latter, students learn that someone discovered π by showing that the ratio C/D is invariant with different-sized circles. It maybe worth dropping the definition of a rhombus to teach the lesson of π.

Finally, assessment will have to change. Multiple-choice, fill-in-the-blank, and even 10-minute problems still will have a place but other assessments will be required. Some of these will be formative assessments that teachers provide as students work through the project or make presentations. As in writing with word processors, multiple drafts will be required. Each will have to be assessed and returned for improvement. That is quite a difference from "you got it wrong, let's move on."

Each of the above will take resources, from political leadership to money. Imbuing all students with quantitative literacy cannot take place without additional instructional materials, substantial teacher training, and new assessment instruments. Why is this investment justified? Because current approaches are failing too many students. Their careers and our society will suffer from it. Teachers in technical college programs complain that students cannot do mathematics. We have enough history to know that students are not going to change. We must, therefore, change what and how we teach mathematics.

Appendix: Empirical Mathematics

Computers take derivatives and integrals, invert matrices for linear programming, and perform other algorithms much better than humans. I have written elsewhere about the end of algorithmic work as a means of making a livelihood in the United States.[34] American workers cannot accept a wage low enough to compete with a computer. Workers do, however, require enough mathematics and creativity to structure a problem so that mathematics can be used. They also need sufficient quantitative capacity to know when tools are not working right or have been improperly used.

This appendix, whose structure is shown in the accompanying table, offers a first stab at a canon of empirical mathematical problems. The table lists mathematical tools for each of the five SCANS competencies and each of the four adult roles. Some of the entries in the table do not include all the tools needed because they have been noted frequently in other boxes. For example, the four arithmetic functions are needed to solve many problems that an American will face, so they are not listed repeatedly. When the phrase "concept of . . . " is used in the table, it means just that—knowing the concept of rate of change and change in the rate of change (acceleration) does not require knowing how to take the second derivative. The same thought applies to linear programming and to the idea that minimums and maximums occur when the derivative is zero or inflection when the second derivative is zero or that an integral takes the sum over an interval.

The main part of the appendix consists of examples of important tasks arranged according to the SCANS taxonomy. The mathematical models described are expected to be developed and expressed in spreadsheets, graphics packages, etc. Students would be asked to estimate "rough numbers" to ensure that the models have been properly specified and the numbers properly entered.

Planning

Budget:

Worker: Using a spreadsheet with algebraic formulas, develop a budget for a retail store, construction project, manufacturing operation, or personal services (e.g., dental) office. The budget should include wages, benefits, material (or inventory), rent, and interest costs on borrowed funds.

Consumer: Using pencil and paper (with a calculator) and given a set of criteria and prices, develop a monthly budget for a family of four. Develop a budget for a party.

Citizen: Given an agency or organization budget for the past five years, write a two-page letter explaining and criticizing it. Include information on the growth or decline of the budget components themselves and as shares of the total. Relate to other variables, such as inflation and population growth.

Personal: Be able to understand the effects of budgets on historical events. Was the Athenian budget for its navy an excessive burden?

Schedule:

Worker: Using a spreadsheet (or other software) with algebraic formulas, develop a schedule for a construction project, advertising campaign, conference, medical regime, or software project. Require conversions from hours to workweeks. Understand the difference between activities done in sequence and simultaneously. Understand PERT and Gantt charts.

Consumer: Using pencil and paper without a calculator, plan a party or a meal. Convert hours to minutes.

Citizen: Understand why it takes so long to build a road or school.

Personal: Appreciate why Napoleon was beaten by the weather and Russia.

Space:

Worker: Using a computer graphics package, lay out a storeroom or office space in three dimensions. Develop a graphic for a brochure. Lay out material for a garment or a steel product. Lay out a restaurant or hotel space. Place paintings in a gallery.

Consumer: Look at a builder's plans and modify them. Understand your own living space.

Citizen: Understand plans for a public building.

Personal: Appreciate good design in products and buildings. Hang paintings in your house.

Staff:

Worker: Using a matrix and database, assign staff to functions. In unusual situations, you may use linear programming to match skills and requirements matrices.

Consumer: Contract out a renovation project for your house.

Citizen: Understand a school's staffing requirements.

Personal: Understand staffing requirements in a historical setting. Assign players in a Little League baseball game or in the local orchestra.

Systems and Processes

Understand:

Worker: Read and understand a flowchart for a production or paperwork process. Read organization charts. Read diagrams explaining how technology functions.

Consumer: Read a flowchart and follow the directions to install a new piece of software or an exercise machine.

Citizen: Read government organization charts. Understand a flowchart for legislation as it passes through Congress.

Personal: Grasp the organization of a military campaign. Understand a diagram explaining how technology functions or a scientific process unfolds. Understand the culture of mathematics and the process of mathematical discovery.

Monitor:

Worker: Use techniques of statistical process control to monitor a manufacturing process or patient or customer complaints.

Consumer: Understand statements about the quality of the products or services purchased.

Citizen: Understand environmental safeguards.

Personal: Monitor changes in a local garden.

Design:

Worker: Develop an information (or other) system flowchart and build a mathematical model to simulate its operation. Develop a statistical process-control system.

Consumer: Design a system for keeping the pipes in a summer house from freezing.

Citizen: Help a local school district design a school safety system.

Personal: Design a system for maintaining a diet and exercise regime.

Interpersonal

Negotiate:

Worker: Negotiate the price of a product or project and be able to think on your feet, including manipulating numbers mentally. Participate in a labor-management negotiation.

Consumer: Be able to understand a construction contractor's or mechanic's proposal and negotiate a fair agreement.

Citizen: Understand a government negotiation.

Personal: Understand a historically important negotiation.

Teach and Learn:

Worker: For teachers, help students do quantitative work in non-mathematical subjects.[35] For workers, teach co-workers or customers the mathematics needed to carry out a task or use a product. Should know enough mathematics to absorb training.

Consumer: Should know enough mathematics to learn how to use a product when taught by a salesperson. Should be able to teach a spouse how to use a product.

Citizen: Should be able to explain and debate policy issues when quantitative issues are involved.

Personal: Should be able to discuss topics when quantitative issues are involved.

Information

Gather and Organize:

Worker: Create a filing system for parts or customer information. Build a database.

Consumer: Create a filing system for tax information. Use a database. Organize an on-line checking system.

Citizen: Use a file to find out about government services in your district. Organize a file for a school's PTA.

Personal: Use a Dewey decimal and an on-line library system to find a book and information.

Evaluate:

Worker: Use a statistical package to evaluate data. Read relevant statistical studies and come to a judgment.

Consumer: Evaluate advertising claims. Read an annual report from a firm whose stock you hold.

Citizen: Evaluate political claims.

Personal: Judge the likelihood of an event or story (UFO) being true.

Communicate:

Worker: Write a report about a quantitative issue, including tables and charts. Make a presentation on the material to more senior colleagues.

Consumer: Read and listen to such reports critically and be able to ask intelligent questions.

Citizen: Make a presentation or write a report for the school board.

Personal: Carry out a conversation about a quantitative issue. Engage in a chat room about a quantitative issue of interest such as astronomy.

Technology

Use:

Worker: Use equipment, such as a numerically controlled machine tool, to produce a part.

Consumer: Use a computer.

Citizen: Use a county's Internet address to find tax data.

Personal: Use a chat room to engage in discussion.

Choose:

Worker: Analyze alternative medical, construction, manufacturing, or computer equipment and recommend a purchase.

Consumer: Analyze alternatives for video on demand, home security systems, or computers.

Citizen: Analyze a county's or school board's decision to purchase technology, from fire engines to computer systems. Be able to judge whether the antimissile system makes sense.

Personal: Analyze a historic technology decision, from the longbow to atomic energy.

Maintain:

Worker: Follow maintenance instructions for a piece of industrial equipment.

Consumer: Follow maintenance instructions for a consumer product.

Citizen: Participate, as part of a volunteer fire department, in maintenance of the fire engines.

Personal: Maintain rare books or valuable paintings when temperature and humidity must be controlled.

Mathematics Required to Solve Frequently Occurring Problems in Four Roles and Five SCANS Competencies[36]

Problem Domains	Planning • *Budget* • *Schedule* • *Space* • *Staff*	Systems and Processes • *Understand* • *Monitor* • *Design*	Interpersonal • *Negotiate* • *Teach and learn*	Information • *Gather and organize* • *Evaluate* • *Communicate*	Technology • *Use* • *Choose* • *Maintain*
Worker Role	Four arithmetic operations, estimation, geometry, algebra, exponential functions, spreadsheets, conversions. Concept of trade-offs. Awareness of tools such as linear programs and calculus for making trade-off decisions.	Model-building. Concept of first and second derivative and of integral, average, and standard deviation.	Mental arithmetic, fractions, percentages.	Create and read graphs, tables, and explanatory text.	Read graphs, tables, and explanatory text, Concept of trade-offs. Geometry.
Consumer Role	Four arithmetic operations, geometry, exponential functions, spreadsheets. Concept of trade-offs.	Concept of first and second derivative and of integral, average, and standard deviation.	Mental arithmetic, fractions, percentages.	Read graphs, tables, and explanatory text.	Read graphs, tables, and explanatory text. Geometry.
Citizen Role	Four arithmetic operations, geometry, concept of trade-offs.	Concept of first and second derivative and of integral, average, and standard deviation.	Mental arithmetic, fractions, percentages.	Read graphs, tables, and explanatory text.	Read graphs, tables, and explanatory text. Geometry.
Personal Role	Geometry, concept of trade-offs.	Concepts of calculus and statistics. History of mathematical discovery.		Read graphs, tables, and explanatory text.	Geometry.

Notes

1. Michael K. Smith. *Humble Pi: The Role Mathematics Should Play in American Education* (Amherst, NY: Prometheus Books, 1994).
2. John Faithful Hamer and Anna-Liisa Aunio, "The Technology Literacy Challenge and the Baltimore Initiative: Integrating School and Work on the Information Highway Final Evaluation Report" (2001).
3. Theodore M. Porter, "The Triumph of Numbers: Civic Implications of Quantitative Literacy," in *Why Numbers Count,* Lynn Arthur Steen, ed. (New York, NY: College Entrance Examination Board, 1997), 1–10.
4. John Dossey, "National Indicators of Quantitative Literacy," in Steen, *Why Numbers Count,* 45–59.
5. "Do You Know the Good News About American Education?" (Washington, DC: Center on Education Policy, 2000): 13.
6. "This Year's Freshmen: A Statistical Profile," *Chronicle of Higher Education*, 28 January 2000, A50.
7. "Do You Know the Good News About American Education?" (Washington, DC: Center on Education Policy, 2000): 27.
8. "This Year's Freshmen: A Statistical Profile," *Chronicle of Higher Education,* 28 January 2000, A50.
9. See http://www.stolaf.edu/other/ql/intv.html.
10. Ibid.
11. As arose in the Bradley-Gore debates in the discussion of rival health insurance premiums.
12. Lynn Arthur Steen pointed out that adding odd fractions is preparation for adding mixed algebraic fractions. The preparation can, in my judgment, wait until students reach such algebra problems (if ever). The cost, in students who become convinced that "mathematics is not for them" is too high to justify the benefit.
13. Robert S. Bernstein and Michele Root-Bernstein, "Learning to Think With Emotion," Chronicle *of Higher Education,* 14 January 2000, A64.
14. An anecdote from Steve Childress of New York University: I just finished grading an exam I gave to graduate students seeking admission to our Ph.D. program. One of the questions I asked (the subject was complex variables) was of a standard kind requiring the calculation of a "residue." Now there are various ways of doing this, certain formulas that are useful in individual cases, but the heart of the matter is that you are seeking a certain coefficient in a series and this can usually be obtained directly by expanding the series for a few terms. My problem could be solved in several lines by this direct approach. I was astounded to see that almost everyone applied a certain formula that, in this problem, led to impossibly complicated mathematics. I asked around about this and learned that we had just instituted a kind of prep course for the exams, and that the instructor had given them a problem of this type and solved it with that special formula (in a case where it worked easily)! This is the crux of the problem I see from day to day, from freshmen on up.
15. Robin Williams, "The Remaking of Math," *Chronicle of Higher Education,* 7 January 2000, A14.
16. Ibid.
17. http://www.stolaf.edu/other/ql/intv.html.
18. Lynn Arthur Steen, "Preface: The New Literacy." In *Why Numbers Count,* Lynn Arthur Steen, ed. (New York, NY: College Entrance Examination Board, 1997), xv–xxviii.
19. F. James Rutherford, "Thinking Quantitatively about Science." In Steen, *Why Numbers Count*, 60–74. Italics in the original.
20. Arnold Packer, "Mathematical Competencies that Employers Expect," In Steen, *Why Numbers Count,* 137-–54.
21. Ibid.
22. David Denby, *Great Books: My Adventures with Homer, Rousseau, Woolf, and Other Indestructible Writers of the Western World* (New York, NY: Simon and Schuster, 1996).
23. We hope that the mathematics canon will be less controversial that Denby's literature canon. On the other hand, that may be an unreasonable hope.
24. See "*What Work Requires of School* (1991) and *Learning a Living* (1992) (Washington, DC: U.S. Department of Labor).
25. http://www.stolaf.edu/other/ql/intv.html.
26. The author of this essay was executive director of SCANS.
27. National Academy of Sciences, *Being Fluent with Information Technology* (Washington, DC: National Academy Press, 1999).
28. I recall my quantitatively literate mother using her skills to figure out when important events—such as births, marriages, and deaths—occurred.
29. Arthur Mattuck, "The Remaking of Math," *Chronicle of Higher Education*, 7 January 2000, A15.
30. I heard one engineering dean wonder if his course in electronics was not the best recruiting tool the School of Business had for transfers to its program.
31. That is, we do not shine even on the disaggregated topics. As to real problem solving, it is not even tested.
32. I, and many others, have been involved in creating CD-ROMs to relieve teachers of the task of project construction.
33. Something advocated by NCTM and emphasized to me by Ivar Stakgold in a private telephone conversation.
34. Hudson Institute, "The End of Routine Work and the Need for a Career Transcript," Hudson Institute Workforce Conference (Indianapolis, IN: September 23–24, 1998).
35. Mathematics teachers would presumably have taken mathematics courses beyond this level.
36. See Appendix for examples of the entries in this table.

Later, they can appreciate mathematics' power to generalize. The inductive approach is more likely to succeed than the current deductive process, in which general rules are taught first and applicationsâ€”selected from the canon noted in Humble Piâ€”are of secondary importance. Do you want to read the rest of this article? Request full-text. Citations (1). Mathematics Stack Exchange is a question and answer site for people studying math at any level and professionals in related fields. It only takes a minute to sign up. Sign up to join this community. Anybody can ask a question. Anybody can answer. The best answers are voted up and rise to the top. Home. Questions.Â So I'd like to know what mathematical proofs you've come across that you think other mathematicans should know, and why. soft-question big-list. share | cite | improve this question |.