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MORPHOLOGICAL PROJECTOR IN THE L_0 METRIC AND THE PROBLEM OF LOCALIZATION OF STRUCTURAL DIFFERENCES BETWEEN IMAGES

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We consider the problem of localization of structural differences between two images given by Borel functions on a bounded planar set. For the case of finite-valued images, we propose a new algorithm for the calculation of the difference domain based on the morphological projection in the L_0 metric. It is shown that the algorithm gives an exact solution for a wide class of structural differences. It turned out that the algorithm based on the morphological projection in L_2 does not give an exact solution in the class of bounded structural changes. For the case of discrete images, when one of them is perturbed by a discrete independent normal white noise, we construct an algorithm for the calculation of the difference domain and show that the symmetric measure of the difference between the algorithm's output and the true difference set vanishes in probability under the unbounded growth of the ratio of the minimum jump to the standard deviation of the noise. We obtain a new estimate for the location of global maximum points for a Gaussian mixture of a special form.

Keywords: morphological analysis of images, morphological projector, Gaussian mixture, metric L_0 , structural changes.

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Problem 18. By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23. 3 7 4 2 4 6 8 5 9 3. That is, $3 + 7 + 4 + 9 = 23$. Find the maximum total from top to bottom of the triangle below: 75 95 64 17 47 82 18 35 87 10 20 04 82 47 65 19 01 23 75 03 34 88 02 77 73 07 63 67 99 65 04 28 06 16 70 92 41 41 26 56 83 40 80 70 33 41 48 72 33 47 32 37 16 94 29 53 71 44 65 25 43 91 52. NOTE: As there are only 16384 routes, it is possible to solve this problem by trying every route. However, Problem 67, is the same challenge with a triangle containing one-hundred rows; it cannot be solved by brute force, and requires a clever method! ;o). Project Euler: Copyright Information | Privacy Policy. summation formula, see 65B15 for: 78: 41-XX Approximations and expansions {}For all approximation theory in the complex domain, see 30E05 and 30E10; for all trigonometric approximation and interpolation, see 42A10 and 42A15; for numerical approximation, see 65Dxx. for: 79: 41Axx Approximations and expansions {}For all approximation theory in the complex domain, see 30E05 and 30E10; for all trigonometric approximation and interpolation, see 42A10 and 42A15; for numerical approximation, see 65Dxx. for: 80: 41A10 Approximation by polynomials {}For approximation by trigonometric polynomials, see 4 UDC: 004.932 MSC: 62M40, 65D18, 68U10 Received: 22.11.2016. Citation: V. B. Kostousov, D. S. Perevalov, "Morphological projector in the L_0 metric and the problem of localization of structural differences between images", Trudy Inst. Mat. i Mekh. UrO RAN, 23, no. 1, 2017, 171-187. Citation in format AMSBIB. \Bibitem{KosPer17} \by V.~B.~Kostousov, D.~S.~Perevalov \paper Morphological projector in the L_0 metric and the problem of localization of structural differences between images \serial Trudy Inst. Mat. i Mekh. UrO RAN \yr 2017 \vol 23 \issue 1 \pages 171--187 \mathnet{http://mi.mathnet