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MORPHOLOGICAL PROJECTOR IN THE L_0 METRIC AND THE PROBLEM OF LOCALIZATION OF STRUCTURAL DIFFERENCES BETWEEN IMAGES

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We consider the problem of localization of structural differences between two images given by Borel functions on a bounded planar set. For the case of finite-valued images, we propose a new algorithm for the calculation of the difference domain based on the morphological projection in the L_0 metric. It is shown that the algorithm gives an exact solution for a wide class of structural differences. It turned out that the algorithm based on the morphological projection in L_2 does not give an exact solution in the class of bounded structural changes. For the case of discrete images, when one of them is perturbed by a discrete independent normal white noise, we construct an algorithm for the calculation of the difference domain and show that the symmetric measure of the difference between the algorithm's output and the true difference set vanishes in probability under the unbounded growth of the ratio of the minimum jump to the standard deviation of the noise. We obtain a new estimate for the location of global maximum points for a Gaussian mixture of a special form.

Keywords: morphological analysis of images, morphological projector, Gaussian mixture, metric L_0, structural changes.

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03C68: Other classical first-order model theory. 03C70: Logic on admissible sets. 03C75: Other infinitary logic.Â 03D65: Higher-type and set recursion theory. 03D70: Inductive definability. 03D75: Abstract and axiomatic computability and recursion theory. 03D80: Applications of computability and recursion theory. 03D99: None of the above, but in this section. 03Exx: Set theory. 03E02: Partition relations.Â 12D10: Polynomials: location of zeros (algebraic theorems). 12D15: Fields related with sums of squares (formally real fields, Pythagorean fields, etc.) 12D99: None of the above, but in this section. 12Exx: General field theory. 12E05: Polynomials (irreducibility, etc.) Morphological Image Processing. n Binary dilation and erosion n Set-theoretic interpretation n Opening, closing, morphological edge detectors n Hit-miss filter n Morphological filters for gray-level images n Cascading dilations and erosions n Rank filters, median filters, majority filters. Digital Image Processing: Bernd Girod, Â© 2013-2018 Stanford University -- Morphological Image Processing 1. Binary image processing. n Binary images are common.Â Caveat: There is another definition of erosion in the literature, which flips the structuring element, as for dilation. The Lagunita online videos use that alternative definition. Matlab function imerode uses the definition on this slide. To the best of our knowledge, there is no such discrepancy defining dilation. In contrast, metric localisation techniques estimate the metric position and orientation of the camera. Tradition-ally, this has been approached by computing the pose from 2D-3D correspondences between 2D features in the query image and 3D points in the model, which are determined through descriptor matching [7, 28, 27, 42, 49]. This as-sumes that the scene is represented by a 3D structure-from-motion model.