

Review of
**I. H. ANELLIS, *VAN HEIJENOORT – LOGIC AND
ITS HISTORY IN THE WORK AND WRITINGS OF
JEAN VAN HEIJENOORT***

Ames, Iowa : Modern Logic Publishing, 1994
xiv + 341pp. ISBN 1-884905-01-3 (hardbound) ISBN 1-884905-00-5 (paper)

JEAN-YVES BÉZIAU

1. INTRODUCTION

Jean van Heijenoort (hereafter JvH) is mainly known among logicians as the editor of *From Frege to Gödel* (hereafter FtG). Most logicians have also heard about his peculiar life which has contributed to turning him into a legendary personage. This aspect became well known after Anita Feferman's popular biography appeared [4].

The book under review is about JvH's work and contains very few spicy details about JvH's personal life. Even the first two chapters (Ch. 1 Curriculum Vitae, Ch. 2 Van Heijenoort at Brandeis) about JvH's life are written only from the perspective of his logical work.

The book includes not only a description of JvH's work in history of logic (Ch. 3 Van Heijenoort as historian of logic) and a philosophical appraisal of foundational issues (Ch. 4 Philosophy and foundations of mathematics) but also a comprehensive exposition of JvH's researches in logic (Ch. 5 Van Heijenoort as logician - Contributions to proof theory), most often ignored, especially because they have not been published. Anellis, who was one of JvH's few PhD students (JvH "supervised only four doctoral theses" [p. 83]), had access to JvH's archives (the book contains a table of contents of JvH's archives as well as a complete bibliography of JvH's works in logic).

Moreover, Anellis's book in fact does not reduce to a book about JvH. There are, together with some critical comments about JvH's vision of history of modern logic, a lot of side remarks on the subject. Therefore, it is highly valuable for anyone who is interested in the history of modern logic.

2. JvH AS A MAN

Anellis tries to demystify the legend surrounding JvH. At the end of his book he quotes this quite phantasmagoric description by one of JvH's former students, Sossinky, just to comment afterwards on how far from any grounded reality it is:

The person who got me back on track was John van Heijenoort (...), a great teacher and an extraordinary personality, whose varied achievements include a doctorate in Paris in functional analysis, fluent knowledge of many languages (including Russian), the design and construction of the first really operative high-fidelity stereo record player, work on radar with Shannon, Wiener and von Neumann during the war and (unbelievable but true) the position of Leon Trotsky's personal secretary at the time of the latter's assassination. [p. 258]

Like Wittgenstein, JvH was not a standard academic logician. During more than ten crucial years of his life he was not working at a university. He left the *Lycée Saint-Louis*, where he was training to enter the *École Normale Supérieure*, at age 20 "to enter full-time into the career of revolutionary" [p. 6]; he started his studies again in 1945 at age 33 at the Graduate School of Arts and Sciences of New-York University, where he received his Master's degree (1946) and his PhD (1949) in mathematics and taught until 1965 before becoming Professor of Philosophy at Brandeis University. After the completion of Anellis's thesis in 1977, JvH retired, and "from 1980 until his death, JvH spent much of his time at Stanford University, where he was busily engaged in work on the Gödel Edition Project" [p. 87].

But even after quitting Marxism, JvH stayed faithful to Trotsky. He worked until 1980 as the supervisor of Trotsky's archives in Harvard and "at the time of his death, he was studying the historical writings of Thucydides in preparation for a comparative study of the ancient Greek's views of history with Trotsky's" [p. 8].

One can say that in some sense the style of life and the personality of JvH during these years in the academic world remain the same as during his life as a secretary and bodyguard of Trotsky in Mexico. In fact he used to go back there often, where he was finally shot by his fourth and fifth wives in 1986.

Nevertheless, JvH was a double faced man. One face is described by A. Feferman and the other by Anellis. It would be interesting to study more closely the connection between the two JvHs. Anellis gives only a few hints:

Van Heijenoort's unique style applied to his mode of expression, at least in the course of lectures, and may even have reflected his political past to the extent that he chose a vivid and forceful phraseology. Thus, for example, he spoke of "killing" quantifiers when talking about quantifier elimination and used "kill" as a synonym for "cancellation" in expressions relating to such things as the cancellation of terms in equations. Formulae which were not well-formed were "junk" and so also, for example, were pathological sets. [p. 86]

According to Anellis, for JvH, life did not reduce to logic. He reports that during a lecture JvH claimed: "Life is not first-order, life is not second order : Life is life." [p. 45]. Probably by so speaking JvH did not intend to assert a tautology. This folksy way of speaking is taken as "extensialist" by Anellis and leads him to write a lengthy footnote about the relations between Heidegger and logic, saying that Heidegger "detested logic" [p. 45]. Such a statement is, at least, ambiguous; for example in the same book which is quoted by Anellis to justify his statement, Heidegger emphasizes the very value of logic for philosophy and says: "we can gain access to philosophy through the concrete problems of logic" [5, p. 7].

In fact it seems that Anellis developed a bad opinion about Heidegger's conception of logic by taking a course with the Heideggerian John Sallis in 1972. Sallis gave a course of logic which was "mechanically taught" [p. 62] and did not speak about the relations between logic and philosophy, and mathematics.

Contrasting with JvH's highly emotional way of speaking is JvH's written style : "Van Heijenoort was always a perfectionist, always sought to be precise, and was always exacting" [p. 201]. In fact, "his fastidious obsession for precision" [p. 166] explains why JvH turned out to be a very respectable historian of logic but also why he didn't become a logician on his own, and did not publish his personal researches in logic.

JvH quickly gained recognition among well established logicians such as Kreisel and Quine through his own idiosyncrasies. As noticed by A. Feferman, Kreisel and JvH had strong affinities, and Anellis tells us that "although JvH was largely self taught in logic and foundations and philosophy of mathematics, he came under the guidance of Georg Kreisel" [p. 28].

Despite having such brilliant friends, JvH was to encounter several difficulties in his own academic life at Brandeis. There he had to face

a man called Frederic Sommers, who “dedicated the greater part of his career to arguing that the history of logic ought to have stopped with Leibniz – or at least before Frege” [p. 50]. For this reason JvH was not able to implement logic at Brandeis:

Van Heijenoort hoped to create a logic program at Brandeis, but his efforts were always thwarted. He attempted to convince the Brandeis administration to appoint at least one other mathematical logician, but his pleas fell on deaf ears. It was in retrospect principally the pervasive influence of Sommers that prevented the philosophy department at Brandeis from becoming the center of research in mathematical logic that van Heijenoort had hoped and expected it would become. [pp. 51-52]

This dispute may explain in part his negative attitude, that we shall speak about later on, towards the algebra of logic and the related tradition.

Anellis informs us that JvH lived at Kirkland Place (Cambridge, Mass.), which was associated with the Peirce family, in particular Charles Peirce stayed there “for several weeks during the late spring of 1879 while in Cambridge to lecture for Harvard’s Philosophy Club on “The Relations of Logic to Philosophy”” [p. 36], and Anellis comments that “it is ironic that JvH, living as he did at 4 Kirkland Place, would have paid so little attention to the work of Peirce” [pp. 104-105].

It seems that JvH’s double face is nothing more than the dichotomy between passion and reason which divides any human being. In some cases, like that of JvH, the contrast is more visible or violent.

JvH definitely quit Marxism after a precise and laborious refutation of Marx’s and Engels’s views on mathematics (cf. his paper of 1948 that he insisted on including forty years later in his *Selected Essays*, probably for emotional reasons). No doubt that it was a rational and radical way to justify the rejection of an exhausted passion. This is how we can interpret the following lines of Anellis:

JvH’s conclusions about the lack of mathematical talents of Marx and Engels, especially Engels, together with their prideful certainty that they understood mathematics better than did professional mathematicians, could not have helped the Marxist case much at a time when JvH was struggling with his doubts about the rectitude of Marxist ideology. [p. 161]

3. JvH AS AN HISTORIAN AND PHILOSOPHER OF LOGIC

According to Anellis:

It was JvH's view that the best way to begin to learn a subject was to understand its history, and that, in turn, could best be done by studying the works of its past masters. [p. 101]

This can explain why JvH, when he started to be interested in logic, became first of all a historian of logic. Generally, nobody thinks that the best way of learning mathematics is to start by learning its history. And nowadays the same prevails for logic. Nobody would start learning mathematical logic by reading the *Begriffsschrift*, then *Principia Mathematica*, then *Grundlagen der Mathematik*, etc. Of course, JvH's idea was not so strange if we think that he began to study mathematical logic at a time in which this discipline was not yet constituted and its history was still young. We can imagine here also a possible influence of Marxism.

Anyway, this historical attitude leads him to edit *From Frege to Gödel* and to become one of the main historians of modern logic. As noticed by Angelelli in his review of the present book in *Mathematical Reviews* [3], Anellis on p.159 claims ambiguously that JvH, in the middle years of our century, was one the two leading historians of logic (*tout court*). Apparently, JvH had little interest in the history of logic before Frege, even in Boole, as we will see. This strange position is perhaps fundamental to understand JvH's view of the history of modern logic and his idea that everything starts *ex nihilo* with Frege.

FtG is unanimously considered to be a great book. It is a titanic and precise work whose result is the gathering of fundamental texts originally published in some journals and languages ordinarily difficult to access and their translations into English, with introductory presentations.

However FtG has been seriously criticized by G. H. Moore in a review of its second edition, whose conclusion is the following:

In sum, we are fortunate to have this source book, particularly for its many translations and excellent introductory comments. Regrettably, the competence found in the translations and introductions was not matched by the choice of selections and the scope of the book [10, p. 470].

Anellis thinks that Moore is right:

Moore's criticism that *From Frege to Gödel* is not a balanced representation of the history of mathematical logic is completely justified [p. 103].

The polemic is related to the two roots of mathematical logic. JvH "admitted the existence of at least two main currents or traditions in the history of logic, namely the Boole-Peirce-Schröder ("algebraic") tradition and the Frege-Peano-Russell (or "quantification-theoretic") tradition" [p. 103], but in fact he always reduced the influence of the first, as noticed by Anellis and Houser:

Contemporary historians of logic, until recently, have either ignored or downplayed the value of the algebraic logic tradition of the nineteenth century (...). JvH was one of the most influential of these historians giving attention to the algebraic tradition only to dismiss it. [2, p. 3]

Probably JvH was also the most influential in establishing the legend according to which Frege is the father of modern logic. An excellent paper by JvH written in 1974, entitled "Historical development of modern logic" and published posthumously by Anellis, starts abruptly like this:

Modern logic began in 1879, the year in which Gottlob Frege (1848-1925) published his *Begriffsschrift*. In less than ninety pages his booklet presented a number of discoveries that changed the face of logic. [7, p. 242]

According to Anellis this rather unilateral and dogmatic view is due to the influence of Russell:

To be fair to van Heijenoort, it is now clear that he was deeply influenced by Bertrand Russell's interpretation of the history of mathematical logic, according to which algebraic logic was a theoretical "dead-end" which could not express most of the mathematics that could be handled with great facility by the quantification theory developed by Frege, Peano, and Russell. [pp. 103-104]

But it seems that this contradicts another statement of Anellis according to which:

Once freed from his youthful dependence upon the domineering personality of Trotsky, van Heijenoort maintained a critical independence, (...) his views on logic, its history – or any topic – were always his own. [p. 28]

Of course, it would be possible to argue in favour of Frege against Boole with serious arguments based on historical facts. But it seems that was not the strategy of JvH, and this is a weak point of his historical work for which he can correctly be criticized. JvH most of the time simply ignored the “algebraic tradition”. As Moore writes “the case by JvH for ignoring Boole and his school was one whose weakness was not redeemed by its brevity” [10, p. 469]. Moore also points out that “Frege’s influence on the history of mathematical logic was limited” [*ibid.*], and that Frege is never quoted, except by Russell, by the other authors of the selected papers of FtG, in contrast to Schröder who is not included in FtG but “appears often, especially in the groundbreaking papers of Löwenheim and Skolem” [*ibid.*].

One reason why JvH took Frege to be the father of modern logic is that he considered him as the founder of quantification theory. But it is known that independently of Frege, Schröder developed quantification theory in the algebraic tradition, and therefore as Anellis rightly notes, the “distinction between the “algebraic” and the “quantification-theoretic” traditions is artificial at best” [p. 103].

JvH’s rejection of the algebraic tradition can also explain the major defect of FtG, the absence of the Polish school of logic:

Of the omission in this source book, one of the most surprising and least justifiable is that of Polish logic. After World War I there arose in Poland a number of fine mathematical logicians such as Stefan (*sic*) Leśniewski, Jan Łukasiewicz, and Alfred Tarski. Although their most important work was done in the thirties, important contributions occurred as early as 1920. To have included this work, however, might have conceded that Gödel’s incompleteness Theorem of 1931 was not, after all, a critical watershed in mathematical logic. [10, p. 470]

The connections between the algebraic tradition and the Polish school are strong. According to J.Woleński,

the first contacts of the Poles with mathematical logic at the academic level took place in 1898, when Twardowski lectured on the new trends in logic, including the algebra of logic as interpreted by Boole and Schröder. (...)

Łukasiewicz’s monograph (1910) on the principle of contradiction in Aristotle (...) included an appendix which was an exposition of the algebra of logic and was

thus the first handbook of mathematical logic in Poland.
[14, p. 82]

As it is known, the theory of logical matrices, which is algebraic in nature, was developed in the twenties in Poland as a basic tool for a general study of propositional logics (on the history of logical matrices, see [9]). Later on, Tarski (cf. [12]) showed how the Whitehead-Russell propositional calculus can be considered as a Boolean algebra. Tarski was very much influenced by Schröder and until the end of his life he kept working in the “algebraic tradition”, as witnessed by [13].

The absence of Tarski in FtG is therefore nothing but logical in view of JvH’s rejection of the algebraic tradition.

Anellis, always fair to JvH, recalls that

To JvH’s credit, he wished to include something of Tarski’s in his [1967a] (i.e. FtG), but Tarski (as recalled by [Quine 1988] - letter to Anellis) declined to give his permission, citing worries over infringements of the copyrights of the publishers of [Tarski 1956]. [2, p. 4] (see also [p. 104])

Of course, one can be fair to JvH on this point. But a question of copyright cannot justify the whole conception of history of logic imposed by FtG.

Anellis claims that:

JvH’s mature interests in philosophy of logic and philosophy of mathematics (...) both stemmed from, and were defined by, his interests in the history of logic. [p. 172]

But it is clear also that JvH’s view of the history of logic is undoubtedly related to a particular conception and philosophy of logic. JvH’s thinking was oriented by the Leibnizian distinction between *lingua characteristica* / *calculus ratiocinator* and the distinction between *logica utens* / *logica magna*, which underlies the difference between *absolutism*, according to which there is only one universal fixed logic, and *relativism*, according to which there may be various systems required for various situations.

Contrary to some recent commentaries by S. Feferman, de Rouilhan and Girard stating that JvH shifted from universalism to relativism, Anellis thinks that JvH always remained universalist and that this distinction didn’t play a relevant role in his conception of the history of logic:

there is nothing implicit or explicit in either JvH's (1974) paper "Historical Development of Logic" or in any of his lectures at Brandeis to indicate that the dichotomy between absolutism and relativism was a theme in his thinking about the history of logic. [p. 180]

In fact if JvH had turned to relativism, making the distinction absolutism/relativism crucial for his view on history, then it would be difficult to understand "JvH's blithe dismissal of the algebraic logicians from Boole to Schröder, the logical 'relativists' of the nineteenth century, as a trivial sidelight to the history of logic" [p. 177].

Anellis writes:

My distinct impression was that van Heijenoort thought that there was only one logic, the mathematical logic of classical first-order quantification theory, and that non-classical logics were extensions of the classical theory and translatable into the terms of classical first-order logic, while higher-order logics are translatable into set theory. [pp. 177-178]

This would explain JvH's acceptance of intuitionistic logic:

for van Heijenoort, however, the fact that intuitionistic logic can be embedded into classical logic as a fragment of classical logic is necessary to enable him to treat it as belonging to logic. [p. 179]

These views are quite ambiguous. It is known that most nonclassical logics are not translatable into classical logic, in fact intuitionistic logic itself is not translatable into classical logic (although it is included in it, not "embedded"), but as proved by Gödel, classical logic is translatable into intuitionistic logic.

Another reason, which seems also not really convincing for admitting intuitionistic logic, is related to JvH's special taste for the tree method:

For JvH, an important test for inclusion as logic was ability to carry out proofs using the tree method. [p. 179]

Maybe the tree method easily applies to intuitionistic logic, but, as proved by Gödel, intuitionistic logic cannot be characterized by a finite matrix; therefore someone who is fond of truth-tables can argue for rejection of intuitionistic logic and the acceptance of many-valued logic.

4. JvH AS A LOGICIAN

JvH first did research in mathematics before focusing on logic and foundations of mathematics. Anellis gives a comprehensive description of the kind of mathematical work JvH was concerned about in his 1949 PhD dissertation *On Locally Convex Surfaces* and explains, as follows, his shift of interest:

Van Heijenoort's interest began to move toward logic and foundations of mathematics and away from classical mathematics in the mid-1950s. This transition developed through the interest in topology, which led to the study of Brouwer's work on topology, which in turn led to a study of Brouwer's work in foundations of mathematics. It was stimulated also by his attendance at the Cornell Summer Institute in Symbolic Logic in 1957, where he took advantage of an excellent opportunity to establish contacts with practicing logicians. [p. 26]

The perspective of JvH's research in logic and its relation with his conception of logic and history of logic is well explained by Anellis:

Van Heijenoort's non-historical work combined his historical insights with respect to the development of quantification theory as a family of formal systems with the concern of elucidating logically the concepts of *consistency*, *completeness*, and *(being a) proof*. Van Heijenoort's non-historical work, largely unpublished, belongs in this sense to the tradition of Hilbertian metalogic as that was elaborated by Hilbert and Bernays in their *Beweistheorie*, as the logical study of proofs of logic. [pp. 203-204]

Most of JvH's logical researches were about trees:

JvH's most important and most original research in logic was, in particular, in the area of model-theoretic proof theory and contributed to the development of the tree method for quantification theory. [p. 204]

JvH was very fond of trees and thought that this method was in particular pedagogically fine. Anellis himself is very interested in trees, JvH's influence we guess, and makes a kind of apology for the tree method which probably reflects JvH's own ideas :

Tree proofs permit graphical-geometric representations of logical relations, and appear to be of greater intuitive accessibility than either the axiomatic method or the method of natural deduction. It combines insights

and results of model theory and proof theory in a fashion that permits exceptionally elegant proofs of the completeness and satisfiability of the method and its variants and clearly identifies the most basic concepts of proof with such model-theoretic results as Craig's Interpolation Lemma, Beth's Definability Theorem and Robinson's Consistency Theorem. [p. 205].

Even defending the tree method, Anellis is not reluctant to quote a valuable criticism of the method:

Marco Mondadori has argued that part of the attractiveness of the tree method, and in particular its reduction of proofs to "quasi-mechanical tests for validity", is a serious handicap insofar as it leads students to believe that "proofs are in general easy to find and that their search is easily mechanizable" and misleads students into confusing proofs and decision procedures. [p. 252].

In order to explain JvH's own contributions to the tree method, Anellis gives a detailed story of the method. It is well summarized as follows:

Thus, the tree method developed from the Beth tableau method which professional logicians such as Jeffrey had found confusing, into a method which has become increasingly popular even with undergraduate philosophy students. It began with Hintikka, who first presented some of the basic ideas of model sets as graphical representations of truth trees of Beth tableaux, and was systematically developed and given its present form by Smullyan, as a left-sided Beth tableau, was first presented to logicians by Smullyan in its full development in 1968, and to logic students in 1967 by Jeffrey in his textbook, while van Heijenoort developed the falsifiability tree as dual of the truth tree presented by Jeffrey and Smullyan, as a modification of the right-sided falsehood tree of the Beth tableau. [p. 255]

Anellis's discussion about trees is very interesting but sometimes difficult to follow, but to be fair to Anellis it seems that the confusion is in the subject itself, especially due to the fact that the tree method mixes model-theoretic and proof-theoretic elements (there indeed lies its interest). Therefore most of the time the terminology is quite ambiguous,

such as the generic term, often used by Anellis, “model-theoretic proof theory”.

Anellis relates that Jeffrey was struggling for understanding Beth tableaux and then met Smullyan in the street who gave him a trick to understand it: “just work with one side – the positive one – and whenever you’d put something on the other side instead just put its denial on the positive side” [p. 218].

According to Anellis, what are nowadays called *Smullyan trees*, originally called by Smullyan *analytic tableaux*, a kind of synthesis of the works of Beth and Hintikka and the work of Hintikka, are generally undervalued in the story. Trees have some older ancestors, Gentzen and Herbrand, and even Lewis Carroll (see [1]).

Anellis claims that “it is easy to show that LK- (i.e. LK without cut) is precisely the tree method” [p. 206]. However, if it is exactly the same method, one can wonder why people like JvH himself have argued that the tree method is better than Gentzen’s. On the other hand, it seems difficult to assimilate sequent calculus with the tree method in the perspective of structural rules.

A first result by JvH about the tree method is reported by Anellis:

One of his contributions was to show that the tree method could be applied to quantified formulae whether or not they were in prenex form. By doing so, he showed that trees need not be finitary. This was done by modifying the character of the quantifier rules of the tree method by applying Herbrand quantification rules for the tree method and by proving König’s infinitary tree lemma. [p. 222]

In a series of unpublished papers, JvH developed his falsifiability tree method. Let us quote the suggestive titles and dates of some of the principal ones (according to Anellis (cf. [p. 204], they possibly will be published in the future):

- “Interpretations, Satisfiability, Validity” (1966)
- “On the relation between the Falsifiability Tree Method and the Herbrand Method in Quantification theory” (1968)
- “Comparison between the Falsifiability-tree Method and Gentzen System” (1973)
- “Falsifiability trees” (1974).

Here is Anellis’s characterization of JvH’s method of falsifiability tree and an interesting result of JvH related to it:

The falsifiability tree, as first proposed by JvH (1968e) as the dual of the Smullyan tree or truth tree, announced

by JvH in (1970a), is a canonization, or algorithmization and codification, of proof by contradiction for LK and for axiomatic methods, and provides a test for the validity of proofs in analytic tableaux and of proofs, as sequences of formulae, in LK-. [p. 225].

One consequence immediate there of his completeness and soundness proofs is the Löwenheim-Skolem theorem, for which he was therefore able to give a one line proof. [p. 236]

It would be interesting to compare the falsifiability tree method with refutation calculi which have been recently worked out by T. Skura (see e.g. [11]) in the spirit of an old idea of Łukasiewicz (see [8]).

5. CONCLUSION

JvH had a tragical personal ending, but this is not the case for his work in the history of logic. His former student, Anellis, who wrote this book about him, pursues his work. As the founder of *Modern Logic*, the first journal exclusively dedicated to the history of modern logic, and author of various historical papers, he continues to carry the torch. Moreover, as it can clearly be seen from the book under review, Anellis is not a blind follower of JvH. Wiser than his master, he has made original contributions, for example, to the history of the algebra of logic.

REFERENCES

- [1] F. Abeles, "Lewis Carroll's method of trees: its origin in Studies in logic", *Modern Logic*, 1 (1990), pp.25-35.
- [2] I. H. Anellis and N. R. Houser, "Nineteenth century roots of algebraic logic and universal algebra", in *Algebraic logic*, H. Andréka, J.D.Monk and I.Néméti (eds), North-Holland, Amsterdam, 1991, pp.1-36.
- [3] I. Angelelli, *Mathematical Reviews*, 96a:03001, 1996, p.16.
- [4] A. B. Feferman, *Politics, logic and love – The life of Jean van Heijenoort*, A.K.Peters, Wellesley, 1993.
- [5] M. Heidegger, *The metaphysical foundations of logic*, Indiana University Press, Bloomington, 1992.
- [6] J. van Heijenoort (ed), *From Frege to Gödel: A source book in mathematical logic (1879-1931)*, Harvard University Press, Cambridge, 1967, 1971 (second edition).
- [7] J. van Heijenoort, "Historical development of modern logic", *Modern Logic*, 2 (1992), pp.242-255.
- [8] J. Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, Clarendon, Oxford, 1951.
- [9] I. Marek, *Początki matryc logicznych*, PhD thesis, Silesian University at Katowice, 1993.

- [10] G. H. Moore, Review of *From Frege to Gödel* (2nd edition), *Historia Mathematica*, 4 (1977), pp.468-471.
- [11] T. Skura, “Refutation calculi for certain intermediate propositional logics”, *Notre Dame Journal of Formal Logic*, 33 (1992), pp.552-560.
- [12] A. Tarski, “Grundzüge des Systemenkalküls. Erster Teil.”, *Fundamentae Mathematicae*, 25 (1935), pp.503-526.
- [13] A. Tarski and S. R. Givant, *A formalization of set theory without variables*, American Mathematical Society, Providence, 1987.
- [14] J. Woleński, *Logic and philosophy in the Lvov-Warsaw school*, Kluwer, Dordrecht, 1989.

NATIONAL LABORATORY FOR SCIENTIFIC COMPUTING, AVENIDA GETULIO VARGAS 333, 25651-070 PETROPÓLIS, RJ, BRAZIL
E-mail address: `jyb@lncc.br`

