JOHN WILDER TUKEY
(1915–2000)

DAVID R. BRILLINGER

> I believe that the whole country—scientifically, industrially, financially—is better off because of him and bears evidence of his influence.
>
> —John A. Wheeler, Princeton Emeritus Professor of Physics

> We have watched at least four Presidents of the United States listen to him and heed his counsel.
>
> —William O. Baker, Retired Chairman of the Board, Bell Labs

John Wilder Tukey (JWT): chemist, topologist, advisor, information scientist, researcher, statistician, educator, data analyst, executive, died of a heart attack on July 26, 2000 in New Brunswick, New Jersey. The death followed a short illness. He was 85 years old.

John Tukey's whole life was one of public service, and as the preceding quotes make clear he had profound influence. He was a member of the President's Scientific Advisory Committee for each of Presidents Eisenhower, Kennedy, and Johnson. He was special in many ways. He merged the scientific, governmental, technological, and industrial worlds more seamlessly than, perhaps, anyone else in the 1900s. His scientific knowledge, creativity, experience, calculating skills, and energy were prodigious. He was renowned for creating statistical concepts and words.

JWT's graduate work was in mathematics, but driven by World War II, he left that field to go on to revolutionize the world of the analysis of data. At the end of the War he began a joint industrial-academic career at Bell Telephone Laboratories and at Princeton University. Science and the analysis of data were ubiquitous. This split career continued until he retired in 1985. Even after retirement his technical and scientific work continued at a very high level.

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In memorializing John Tukey's life one has to review his contributions to both science and society. It is further interesting to follow his evolution from mathematician to statistician. The sections of this article are: Narrative of the Professional Life, The Scientific Legacy, Concluding Remarks, and Epilogue. Side panels list his Ph.D. students and his books.

Narrative of the Professional Life

Chronology. John Tukey was born in New Bedford, Massachusetts on June 16, 1915. After being schooled precollege at home, he earned bachelor's and master's degrees in chemistry. For a Ph.D. he went to Princeton, where he quickly switched to mathematics.

Graduate School. He arrived at Princeton in September 1937. One has the strong impression that John and his friends had a wonderful time during their graduate studies. The Graduate College and Fine Hall, where the Mathematics Department was located, were very special places. Meals in the Graduate College were taken in academic gowns, and eating groups formed. In the first year JWT was drafted into the "Fuhrocracy" whose members included L. Spitzer, F. Smithies, R. P. Boas, G. W. King, M. Kanner, H. Wallman, and A. Stone. In later years his group included G. W. Brown, Richard Feynman, and O. Morgenstern. His other friends included N. Steenrod, B. MacMillan, and Bill Baker.

Fine Hall, besides housing the Mathematics faculty, was renowned for its library and for tea time in the Lounge. The Princeton Mathematics faculty at that time included: C. Eisenhart, S. Lefschetz, S. S. Wilks, and A. W. Tucker. The Members of the Institute for Advanced Studies (IAS), such as John von Neumann, were also housed in Fine Hall then.

JWT attended many (all?) mathematics lectures. Of those by Marston Morse on symbolic dynamics, he said that Steenrod and he did their best to keep "Morse honest". Keeping people honest remained a key part of his scientific life. Of a course A. W. Tucker gave in 1938, Tucker once said that every time he gave a definition of a combinatorial manifold, JWT would come up with a counterexample. Tucker further remarked, "It ended in a draw."

At the party celebrating the completion of their graduate studies, JWT and Robert Eddy became famous for serving milk rather than the traditional beer. In a story whose authenticity was not denied by JWT, John von Neumann is reputed to have remarked, "there is this very bright graduate student, and the remarkable thing is that he does it all on milk."

While living in the Graduate College, John came to know the physicist Richard Feynman, and he appears in various of the books by and about Feynman. One special story relates to keeping time. Feynman knew that he could keep track of time while reading, but not while speaking. He presented this as a challenge. Rising to it, JWT showed that he could speak and keep track of time simultaneously. Of this Feynman remarks: "Tukey and I discovered that what goes on in different people's heads when they *think* they're doing the same thing—something as simple as *counting*—is different for different people." This may also be the source of JWT's remark, "People are different."

In 1939, Feynman and Tukey, together with Bryant Tuckerman and Arthur Stone, were members of the Flexagon Committee. This group formed directly following the discovery of certain origami-like objects by Stone. Flexagons are folded from strips of paper and reveal different faces as they are flexed. A theory of flexagons was worked out by Tukey and Feynman, the theory being a hybrid of topology and network theory. Feynman created a diagram that showed all the possible paths through a hexaflexagon. The Feynman-Tukey theory was never published, but parts were later rediscovered.

World War II. In 1941 Tukey was appointed Assistant Professor of Mathematics at Princeton. That year he listed his interests in *American Men of Science* as point set topology and analysis. With America's entry into World War II, things changed dramatically. John joined the Fire Control Research Office, at 20 Nassau Street, Princeton. This organization was concerned with armored vehicle fire control, ballistic behavior of rocket powder, gunfire control, and the accuracy of anti-aircraft stereoscopic height and bomber range finders.

While working there JWT had countless interactions with Charles P. Winsor (1895–1951), a Harvard engineer with a Ph.D. in physiology. He was referred to as an "engineer-turned-physiologist-turned-statistician." To quote John: "It was Charlie and the experience of working on the analysis of real data that converted me to statistics." JWT encountered many others at 20 Nassau Street who went on to renowned careers in statistics. It is hard to know just what the research consisted of, but one finds the following "Example" in the Mosteller and Tukey regression book:

> During World War II, in investigating aiming errors made during bomber flights over Europe, one of the research organizations developed a regression equation with several carriers (explanatories). Among its nine or so carriers were altitude, type of aircraft, speed of

> the bombing group, size of group, and the amount of fighter opposition. On physical grounds, one might expect higher altitudes and higher speeds to produce larger aiming errors. It would not be surprising if different aircraft differed in performance. What the effect of size of group might be can be argued either way. But few people will believe that additional fighter opposition would help a pilot and bombardier do a better job. Nevertheless, amount of fighter opposition appeared as a strong term in the regression equation—the more opposition, the smaller the aiming error. The effect is generally regarded as a proxy phenomenon, arising because the equation had no variable for amount of cloud cover. If clouds obscured the target, the fighters usually did not come up and the aiming errors were ordinarily very large.

Bell Telephone Laboratories and Princeton. In early 1945 "Mr. Tukey" began his long association with Bell Labs, Murray Hill, perhaps the foremost industrial research organization the world has known. He never left. He was full time at the Labs for a very brief period, taking leave from Princeton, but quickly switched to half-time at Princeton and half-time at the Labs. At the Labs, he was in succession Member of Technical Staff, Sub-Department Head of Statistics Research, Associate Executive Director Communication Principles Research, and Associate Executive Director Information Sciences.

In his parallel career at Princeton, John rose through the ranks to become Full Professor of Mathematics, a position he held 1950–1966. He was first Chair of the new Statistics Department 1966–1970, and Professor of Statistics from 1966 until his retirement in 1985.

At the Labs, JWT was involved in a myriad of projects. One such project was the Nike missile system, in whose research and development the Bell System played a very substantial role. B. D. Holdbrook and JWT did the aerodynamics, the trajectory, and the warhead design. A collaboration with Ralph Blackman led to two highly influential papers that introduced the engineering and scientific worlds to the computational practice of spectrum analysis. The Labs supported much of his more academic research, as he has acknowledged.

In 1956, Tukey set up the Statistical Techniques Research Group (STRG) at Princeton. The first computer on the Princeton campus was very possibly the IBM 650 at STRG's Gauss House on Nassau Street. Calculation and computers were ever present in JWT's life. The Fire

Control Office had one of the first IBM multiplying punches and used it creatively. The classic work by Burks, Goldstine, and von Neumann (1946) states in the Preface: "The authors also wish to express their thanks to Dr. John Tukey, of Princeton, for many valuable discussions and suggestions." Of just what JWT did Arthur Burks writes:

> John Tukey designed the electronic adding circuit we actually used in the IAS Computer. In this circuit, each binary adder fed its carry output directly into the next stage without delay. ... And this was the circuit actually used because it was reliable and much simpler than the alternative.

Retirement. JWT retired from both the Labs and Princeton in 1985 at age 70. At the retirement event W. O. Baker, Retired Chair of the Board, remarked:

> John has had an incisive role in each major frontier of telecommunications science and technology: uses of transistors and solid state; digital coding and computers; ...; evolution of software and operations support systems; earth satellite and other microwave techniques; electronic switching; laser-based photonics; topology of integrated circuits; ....

At Princeton, JWT became Emeritus Donner Professor of Science and Senior Research Statistician. He further worked as a consultant at places including: Xerox PARC, Bellcore, Merck, the Health Effects Institute, and the Educational Testing Service (ETS).

Contributions to Society. Through the Bell Labs, Princeton, and Washington connections Tukey's skills became well known to many, and there were countless calls on his time. Throughout the years, he chaired or was a member of countless national panels, particularly ones concerned with protecting the environment. Here I mention some of his most notable activities.

The Kinsey Report. In 1950, following a request, the American Statistical Association assembled a committee to review the statistical problems in Alfred C. Kinsey's work on sex research, work described in his controversial 1948 book, *Sexual Behavior in the Human Male.* The members of the committee were W. G. Cochran, F. Mosteller, and JWT. They were concerned particularly with the sampling methods and the absence of controlled randomness.

Things did not go well in the committee's interaction with Kinsey. For example, following a dinner at the Kinsey home, Kinsey's wife, Clara, said "I never fed a group of men that I would have so liked to have poisoned. ... Tukey was the worst." This and other anecdotes may be found in the J. H. Jones biography *Alfred C. Kinsey*.

The U-2 Airplane. The "Cold War" was in full flight in the middle 1950s. One concern was the existence of a "bomber gap" that might enable the USSR to launch a surprise attack. Spurred by such concerns, President Eisenhower created the Killian Committee in 1954. Its Project 3 concerned technical intelligence collection. It was chaired by E. H. Land of Polaroid camera fame and included the astronomer J. G. Baker, the chemist J. W. Kennedy, the engineer A. Latham, Jr., the Nobel laureate physicist E. M. Purcell, and JWT. Project 3 recommended the adoption of "a vigorous program for the extensive use ... of the most advanced knowledge in science and technology." The end proposal was to create a high-flying spy plane, and the development of the U-2 followed very quickly. Photos obtained by overflights of the USSR refuted the claims of a bomber gap, but at the same time they gave evidence of efforts to build long-range missiles. I heard it said that it was JWT who suggested the use of titanium in the construction of the bodies of such airplanes.

In 1957, Eisenhower formed the President's Scientific Advisory Panel partly in response to the effective way that the scientists had dealt with the bomber gap.

Nuclear Disarmament. In 1959, JWT spent a month in Geneva, Switzerland as a U.S. Delegate to Technical Working Group 2 of the Conference on the Discontinuance of Nuclear Weapon Tests. His expertise, in part, concerned the time series problem of distinguishing earthquakes from nuclear explosions. John's wife, Elizabeth, said once that JWT was the one who suggested that tests might be able to be masked to a degree. This possibility very much startled the Russians.

JWT has spoken about the difficulties of that period, describing it as a "time of stress ... like a utilities rate case." In 1967 he wrote:

> ... resembles my experiences with Geneva ... where the political conference set up technical working groups in the hope that the scientists would settle some of the questions that the politicians could not. ... The politicians' attempt to evade responsibility failed, as was inevitable.

Psephology—Election Forecasting. In 1960, RCA/NBC hired a statistical consulting firm to develop a procedure for projecting election results on the basis of partial counts. They involved John Mauchly and JWT amongst others. John became renowned that year for preventing an early call of victory for Richard Nixon.

Starting with the 1962 Congressional election, John assembled a statistical team to develop required methodology and to analyze the results as they flowed in. In the development of methods, the uncertainty was just as important as the point estimates. It turned out that the problem of projecting turnout was more difficult than that of projecting candidate percentage. The procedures developed might be described as an early example of empirical Bayes, or "borrowing strength", to use JWT's term.

NBC stopped involving John in 1980. One reason was that exit polls, where people are interviewed after they leave a polling station, had gotten refined. One can speculate on what might have happened had JWT been involved with one of the networks in making the Florida projections in the fall of 2000.

Education. JWT was very interested in the effectiveness of education. From 1965 on he was a consultant to ETS in Princeton. He had a long involvement with the National Assessment of Educational Progress (NAEP), which is a long-term program of assessing what the inhabitants of the U.S. can do as measured by their performance on exercises at various ages. JWT, Lee Cronbach, Ralph Tyler, Bob Abelson, and Lyle Jones met often in the late 1960s to guide its formation. JWT brought robust procedures, novel ways to measure change, uncertainty estimation, and careful interpretation of the results into the work.

Information Retrieval. Starting in the mid-1960s, John Tukey sought to bring order to the literature of statistics and probability by constructing indexes of the papers of those fields. He had done extensive work for *Mathematical Reviews* and prepared bibliographies before, e.g., for time series, and perhaps this is what spurred him on. In particular he constructed a citation index. Regarding that effort it is impressive to see the roll call of eminent statisticians that JWT recruited to compile the papers and the reference lists. The citation index that was constructed then was one of the earliest outside of the legal profession. JWT constructed other indices as well. These were taken over later by the American Mathematical Society and are part of MathSciNet.

After retirement JWT consulted at Xerox PARC. His name, with a variety of co-inventors from that company, appears on quite a number of patents, some of the key words of which are: *information access, ordering document clusters, identifying drop words, generating thematic summaries, automatic document summarization, phase query formation.*

Census Adjustment. JWT advised the Census Bureau for many years. In 1980 the Census Bureau's step of adjusting the raw values to obtain "improved" estimates became a political issue. Probably to the great surprise of the Bureau, it found itself tangled up in the adversarial setup of the U.S. legal and political systems. This happened because congressional apportionment and the allocation of funds are both based on census counts. The Constitution states, regarding the House of Representatives, "The actual Enumeration shall be made ... in such Manner as they shall by Law direct." This leaves lots of room for argument.

JWT felt that adjustment should be made because errors would be smaller and their direction unpredictable. Regarding the idea that the errors would be smaller, he was perhaps spurred by the success of the borrowing strength procedure in election forecasting.

Teaching. JWT's teaching style was on the oblique side. Indeed, sometimes people wondered if he was being deliberately obscure. It has been suggested that his home schooling background led to some of the difficulties. My personal impression was that he wanted the people with whom he interacted to figure things out on their own, to the degree possible.

The courses he presented at Princeton were state of the art, indeed introducing the art in many cases. The topics included: Monte Carlo, fractional replication, time series, ... Major investigators, such as Cuthbert Daniel, came along to class.

John's interactions with students often took place at the house on Arreton Road, particularly on Saturdays. In particular he believed that gardening was a social activity.

John was always very busy. In one period of his life, to deal with obligations in Washington and elsewhere, he scheduled four classes each week. Then he picked the three times that were the most convenient for the week at hand. Another method was to schedule classes for, say, 2–4 Tu/Th instead of the usual 2–3:15 PM, and to skip a class sometimes and at other times to run it for two hours.

The Scientific Legacy

Mathematics. JWT's contributions to mathematics proper were pre World War II. They evidenced the mathematical ability that stood him in such good stead in his statistical and scientific work.

His thesis, "On Denumerability in Topology", was submitted in 1939, nominally under the supervision of Lefschetz. Part of it later appeared in 1942 in the *Annals of Mathematics Studies* series under the title *Convergence and Uniformity in Topology.* Chapter 3 was never published because of overlap with work of M. M. Day.

The principal part of the thesis was a formulation of uniformity in topology. JWT was seeking to extend convergence techniques to general spaces. The structures have three distinct, but equivalent, definitions via: Weil's pseudometrics, Bourbaki's entourages, and Tukey's families of covers. Appearing in the thesis is the so-called Teichmüller-Tukey Lemma: "every nonempty collection of finite character has a maximal set with respect to inclusion." This lemma is equivalent to the Axiom of Choice. Also in his book JWT christens Zorn's Lemma.

Tukey's approach was used extensively by J. R. Isbell and N. R. Howse in their books. Howse recently wrote me:

> ... But Tukey's contribution in this area went beyond showing us how we should think about uniform spaces. His insight was almost prophetic. He sensed that the most interesting uniform spaces were the ones that were fully normal. ... Tukey predicted that the fully normal uniform spaces would play a major role in mathematics, and indeed they have.

D. H. Fremlin wrote:

> Of course Tukey's ideas on partial orders did have great influence on me. This was really through J. R. Isbell. Steve Todorovic took the same ideas much further ... I still believe that Tukey's notion of cofinal equivalence gives fundamental insight into some important questions in set-theoretic analysis.

There are papers with R. P. Boas and with A. Stone as well as solo-authored articles. The one with Stone, titled "Generalized sandwich theorems", showed that the volumes of any n solids in $\mathbf{R}^n$ can be simultaneously bisected by an $(n-1)$ sphere, with a plane able to be regarded as a sphere of infinite radius. The Ham Sandwich Theorem is the case of $n = 3$.

Statistics.

> Statistics is a science, not a branch of mathematics, but uses mathematical models as an essential tool.
>
> —JWT

The Mosteller-Tukey bombing example mentioned above gives an indication of the difficulties of interpretation of results that statisticians deal with regularly in the world of experiments and data. JWT's statistical contributions reflect his struggles with that often messy world. Some highlights follow.

Robustness. Robustness refers to the property of a procedure remaining effective even in the absence of usual assumptions such as normality and no incorrect data values. In simplest terms the idea is to improve upon the use of the simple arithmetic average in estimating the center of a distribution. As a simple case one can ask: is it ever better to use the sample median than the sample mean, and if so, when?

Tukey had gotten interested in the problem of robustness in the last stages of his work at 20 Nassau Street. He remarked that C. P. Winsor taught him to beware of extreme deviates. The two learned, for example, that if you add 0.1 per cent of a Gaussian three times as spread out as a basic Gaussian, then the mean deviation does better than the classic standard deviation as a way of measuring scale.

An important part of JWT's contribution was developing particular forms of problem distributions, those displaying the difficulties that occurred in practice. The ones employed included the mixture of Gaussians, the slash, and the g-h.

For the smoothing case he recommended running medians. This tool has proved exceedingly useful in image processing.

Time Series. During World War II, JWT had become acquainted with Norbert Wiener's seminal work, later published in book form.

JWT has told the story of how he got interested in the spectral analysis of time series. Just after the War, a Bell Laboratories engineer working on tracking radars was heading to a conference and wished to show a slide of an estimated power spectrum. He met with Tukey and Richard Hamming. Those two knew the Fourier relation between the autocovariance function and the spectrum and took the Fourier transform of the sample autocovariance function. The estimate was seen to ripple. This led Hamming to make the suggestion that the engineer's estimate would look better were it smoothed, for example via the weights 0.25, 0.50, 0.25. The result was a striking success and led to JWT's major involvement in the field of time series analysis.

Tukey often argued the advantages in favor of the power spectrum over the autocorrelation. Such an argument is needed because the two are mathematically equivalent. He claimed that the analyst could learn more from a spectral analysis, particularly in discovering unexpected phenomena.

Bogert, Healey, and Tukey introduced cepstral analysis as a way to address the problem of distinguishing an earthquake from an explosion [1]. This was based on the observation that a signal laid on top of itself with a delay leads to a rippling spectrum. The delay may be estimated as the frequency of the ripple and from that the depth of the seismic event. A lot of colorful language, like saphe cracking, quefrency, and alanysis was also introduced in that paper. Of the language Dick Hamming remarked to John that "from now on you will be known as J. W. Cutie". The cepstrum was quite unlike any time series quantity that had been proposed previously.

Nearly all of the time series papers may be found in Volumes I and II of the *Collected Works*.

Data Analysis. JWT had a philosophy for studying experimental results from early on. He called it data analysis. One of his descriptions was the following:

> . . . data analysis which I take to include, among other things: procedures for analyzing data, techniques for interpreting the results of such procedures, ways of planning the gathering of data to make its analysis easier, more precise or more accurate, and all the machinery and results of (mathematical) statistics which apply to analyzing data.

His 1962 paper on the subject changed the paradigm and language of statistics. His 1977 book made the techniques available to a very broad audience. JWT recognized two types of data analysis: exploratory data analysis (EDA) and confirmatory data analysis (CDA). In the former the data are sacred while in the latter the model is sacred. In EDA the principal aim is to see what the data are "saying". It is used to look for unexpected patterns in data. In CDA one is trying to disconfirm a previously identified indication, hopefully doing this on fresh data. It is used to decide whether data confirm hypotheses the study was designed to test.

Some parts of EDA are ugly, but the real world is ugly, particularly when errors and other aberrant material enters a data set.

Statistical Graphics.

> The best single device for suggesting, and at times answering, questions beyond those originally posed is the graphical display.
>
> —JWT

E. Tufte has remarked of Tukey's work on graphics that "He made the field respectable." In particular one can mention JWT's paper at the Vancouver International Congress of Mathematicians.

Statistical graphics are a critical part of EDA. Tukey's boxplots and stem-and-leaf diagrams now appear throughout scientific presentations and high-school texts.

A field that he popularized is dynamic graphics. Following Tukey's direction a display program, PRIM-9, was developed at SLAC (Stanford Linear Accelerator Center) for looking at multidimensional data. It was based on point cloud rotation and associated dynamic operations. Nowadays point cloud rotation is basic to statistical packages. One difficulty was which direction to rotate to. JWT proposed the procedure of projection pursuit to handle this. It seeks 1 and 2 dimensional projections of multivariate data looking for data points that concentrate near intermediate-dimensional manifolds.

JWT's last book, written together with Basford, was titled *The Graphical Analysis of Multivariate Data*. It contained novel graphical procedures and his current views on the problem of multiple comparisons.

Analysis of Variance (ANOVA). ANOVA and regression are the workhorses of statistics, and JWT made many important contributions to each of them. An assumption of additivity is basic, and JWT was concerned with how to examine that assumption in practice. He remarked "I carried the problem about how to do something about removable non-additivity 'in my pocket' for at least two years before the right idea dawned". The idea was to work with a bilinear expansion in the additive parts. The resulting paper was titled "One degree of freedom for additivity".

Another novel idea of his was dyadic ANOVA, an analysis of variance for vectors. To develop a multivariate analysis of variance he worked with the inner product of the data vector with an abstract-valued vector of the same length.

Multiple Comparisons. In many statistical situations there is a concern to control the error rate of statements being made. Difficulties arise if one gives into temptations associated with the application of a number of tests to the same data set, as in asking several questions at the same time, or in carrying out several analyses of the same data set.

This is another problem that JWT acknowledged working on for a number of years before developing a solution. He actually commented once on how few papers he had written during that period. His solution was developed in a famous 1953 unpublished manuscript, "The Problem of Multiple Comparisons". It now appears in the *Collected Works*.

The Tukey technique is often seen under the rubric of the "Honestly Significantly Difference" (HSD) test, another term that is JWT's. In later years JWT moved on to using the False Discovery Rate (FDR) of Benjamini and Hochsted when many things were being compared.

The Jackknife. This is a tool that Tukey proposed and used for CDA over many years. In naive terms it provides an indication of the uncertainty of an estimate by judiciously combining estimates based on subsets of the full data set.

He called the procedure the jackknife because

> The procedure ... shares two characteristics with a Boy Scout Jackknife:
> (1) wide applicability to very many different problems, and
> (2) inferiority to special tools for those problems for which special tools have been designed and built.

It provides uncertainty analyses when exact estimates are unavailable.

Other contributions. JWT successfully pushed for applications of statistical methods to a wide variety of fields. To mention a few: economics, aeronautics, geophysics, oceanography, meteorology, space science, and astrophysics.

Brief mention may also be made of JWT's novel contributions to: mathematical statistics, quality assurance, theory of games, and medical statistics. There is much that remains unpublished, including various Princeton course notes, working papers, and technical reports. It is a shame that work ended on his *Collected Works*, which brought various unpublished items into public view.

Computation.

Head/hand methods. The arithmetic skills of Feynman and von Neumann are legendary, and Tukey had these too. In particular he developed a variety of quick methods for carrying out statistical analyses in real time. These were basic to his consulting work. He spoke of "using pen, paper, and slide rule". There are a variety of his numerical short cuts presented in the EDA book.

JWT was a member of the Supervising Scientific Committee of the highly influential handbook of mathematical functions edited by Abramowitz and Stegun. The story was the suggestions were all generated spontaneously during a single lunch meeting.

Monte Carlo/Simulation. JWT made contributions to the techniques of Monte Carlo. In 1956, Trotter and Tukey introduced the technique of conditional Monte Carlo wherein one estimates an integral of a weighted function by simple Monte Carlo, instead of the integral of the function itself. This is a means of variance reduction. In it one imbeds the sample space in a larger one.

He also introduced polysampling wherein a single set of samples is able to produce estimates of means and standard errors under two, three, ... different distributions at the same time by combining the sample with two, three, ... sets of weights.

Fourier Analysis. JWT had picked up on the Gibbs phenomenon and the practical need for convergence factors early on. He also wrote concerning the difficulties of interpretation such as those caused by aliasing.

In lectures at Princeton in 1963, he showed that if one wished the discrete Fourier transform of a stretch on N numbers, then when $N = GH$ the computation of the empirical Fourier transform required but $(H + 2 + G)GH$ multiplications. He further remarked that for $N = 4^k$ one needed fewer than $2N + N \log_2 N$. The resulting Cooley-Tukey paper, [3], has become a citation classic. When it popularized the FFT, signal processing very quickly switched from analog to digital.

There is one sad aspect to the FFT story. On numerous occasions JWT made remarks of the type "Gordon Sande ... had purely independently found an algorithm to reach the same effect". He clearly felt bad about Sande's not receiving some share of the credit.

Another side of Tukey's FFT work was his and his collaborators' finding novel applications of the algorithm(s).

Concluding Remarks

Support and Recognition. Throughout his career JWT served in various capacities as an officer in professional societies: AMS Council, Vice-President ASA (American Statistical Association), Vice-President

SIAM (Society for Industrial and Applied Mathematics), President Institute of Mathematical Statistics. He received many prominent honors: Member National Academy, Recipient National Medal of Science, Foreign Member Royal Society of London, and some seven honorary doctorates. He received his first from Case Institute of Technology in 1962. The citation of the award from the University of Waterloo in 1999 included the words: "He has pioneered developments in fields that intersect with every development in mathematics facilities."

Tukey Neologisms. John Tukey was famous for creating new words and new uses for old ones. He said that he did this to avoid confusions.

He is said to have introduced the terms: "bit", "linear programming", "ANOVA", "Colonel Blotto", and was first into print with "software". Of these efforts L. Hogben and M. Cartwright wrote "The introduction by Tukey of bits for binary digits has nothing but irresponsible vulgarity to commend it." Tukey's word polykay was described as "linguistic miscegenation" by Kendall and Stuart because of its combining a Greek prefix with a Latin suffix. JWT did it again later with "polyspectrum".

His own name led to some amusing confusions. In 1980, NASA sponsored a symposium on using robotics to explore space using self-replicating systems. (Feynman and JWT were among the attendees.) Coming out of the discussion was "Tukey's ratio"—the fraction of total necessary resources that must be supplied by some external agency. It was also referred to in that report as the "Turkey Ratio".

The Collected Works. Many of the Tukey papers, as well as a number of previously unpublished works, may be found in *The Collected Works of John W. Tukey* published by Wadsworth, Belmont. These volumes were Bill Cleveland's wonderful prescient idea. There are JWT's and Editors' Forwards discussing various of the individual papers.

There are a partially complete curriculum vitae, a list of coauthors, and a bibliography of his works in [2]. Papers are still appearing. JWT has more than 105 co-authors, the most joint works being with Fred Mosteller. There is a Bell Labs web site containing a variety of materials related to John, including personal reminiscences [4]. There is an oral history with JWT and others mentioning him [5].

Epilogue

John Tukey was a giant of a scientist and public servant. He was an academic who liked to argue and expected to win. But at the same time he was the most generous, patient, caring soul.

When my elder son died after a twenty-year struggle with a brain tumor, and JWT heard of his death, he telephoned. He was weeping away.

JWT dealt with his own personal grief when his wife Elizabeth died; he wrote some very special words:

One is so much less than two.

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I apologize for not referring more specifically to JWT's collaborators on occasion. JWT himself was totally generous in acknowledging them.

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Ph.D. Students of John W. Tukey

Fredrick Mosteller 1946
John Walsh 1947
Donald Fraser 1948
Melvin Peisakoff 1950
Leo Goodman 1950
Bernard Sherman 1950
Ray Murphy 1951
Paul Meier 1951
Alan James 1953
David Wallace 1953
Marvin Minsky 1954
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John Wilder Tukey. Born: 16 June 1915 in New Bedford, Massachusetts, USA Died: 26 July 2000 in Princeton, New Jersey, USA. Click the picture above to see two larger pictures.Â John Tukey's parents, Ralph H Tukey and Adah M Tukey, recognised that John had great potential while he was still only a child, so they arranged for him to be educated at home rather than in school. This was possible since his parents were themselves secondary school teachers who were trained in classics.