APPLICATIONS OF CLASSICAL PHYSICS

Roger D. Blandford and Kip S. Thorne

version 1200.1.K.pdf, January 28, 2013

Preface

Please send comments, suggestions, and errata via email to kip@caltech.edu, or on paper to Kip Thorne, 350-17 Caltech, Pasadena CA 91125

This book is an introduction to the fundamentals and 21st-century applications of all the major branches of classical physics except classical mechanics, electromagnetic theory, and elementary thermodynamics (which we assume the reader has already learned elsewhere).

Classical physics and this book deal with physical phenomena on macroscopic scales: scales where the particulate natures of matter and radiation are secondary to the behavior of particles in bulk; scales where particles' statistical as opposed to individual properties are important, and where matter's inherent graininess can be smoothed over. In this book, we shall take a journey through spacetime and phase space, through statistical and continuum mechanics (including solids, fluids, and plasmas), and through optics and relativity, both special and general. In our journey, we shall seek to comprehend the fundamental laws of classical physics in their own terms, and also in relation to quantum physics. Using carefully chosen examples, we shall show how the classical laws are applied to important, contemporary, 21st-century problems and to everyday phenomena, and we shall uncover some deep connections among the various fundamental laws, and connections among the practical techniques that are used in different subfields of physics.

Many of the most important recent developments in physics—and more generally in science and engineering—involve classical subjects such as optics, fluids, plasmas, random processes, and curved spacetime. Unfortunately, many physicists today have little understanding of these subjects and their applications. Our goal, in writing this book, is to rectify that. More specifically:

- We believe that every masters-level or PhD physicist should be familiar with the basic concepts of all the major branches of classical physics, and should have had some experience in applying them to real-world phenomena; this book is designed to facilitate that.

- A large fraction of physics, astronomy and engineering graduate students in the United States and around the world use classical physics extensively in their research, and even more of them go on to careers in which classical physics is an essential component; this book is designed to facilitate that research and those careers.

- Many professional physicists and engineers discover, in mid-career, that they need an understanding of areas of classical physics that they had not previously mastered. This

book is designed to help them fill in the gaps, and to see the relationship of topics they study to already familiar topics.

In pursuit of these goals, we seek, in this book, to *give the reader a clear understanding of the basic concepts and principles of classical physics.* We present these principles in the language of modern physics (not nineteenth century applied mathematics), and present them for physicists as distinct from mathematicians or engineers — though we hope that mathematicians and engineers will also find our presentation useful. As far as possible, we emphasize theory that involves general principles which extend well beyond the particular subjects we study.

In this book, we also seek to *teach the reader how to apply classical physics ideas.* We do so by presenting contemporary applications from a variety of fields, such as

- fundamental physics, experimental physics and applied physics,
- astrophysics and cosmology,
- geophysics, oceanography and meteorology,
- biophysics and chemical physics,
- engineering, optical science & technology, radio science & technology, and information science & technology.

Why is the range of applications so wide? Because we believe that physicists should have at their disposal enough understanding of general principles to attack problems that arise in unfamiliar environments. In the modern era, a large fraction of physics students will go on to careers away from the core of fundamental physics. For such students, a broad exposure to non-core applications will be of great value. For those who wind up in the core, such an exposure is of value culturally, and also because ideas from other fields often turn out to have impact back in the core of physics. Our examples will illustrate how basic concepts and problem solving techniques are freely interchanged between disciplines.

Classical physics is defined as the physics where Planck's constant can be approximated as zero. To a large extent, it is the body of physics for which the fundamental equations were established prior to the development of quantum mechanics in the 1920's. Does this imply that it should be studied in isolation from quantum mechanics? Our answer is, most emphatically, "No!". The reasons are simple:

First, quantum mechanics has primacy over classical physics: classical physics is an approximation, often excellent, sometimes poor, to quantum mechanics. *Second*, in recent decades many concepts and mathematical techniques developed for quantum mechanics have been imported into classical physics and used to enlarge our classical understanding and enhance our computational capability. An example that we shall discuss occurs in plasma physics, where nonlinearly interacting waves are treated as quanta ("plasmons"), despite the fact that they are solutions of classical field equations. *Third*, ideas developed initially for "classical" problems are frequently adapted for application to avowedly quantum mechanical subjects; examples (not discussed in this book) are found in supersymmetric string theory and in the liquid drop model of the atomic nucleus. Because of these intimate connections

between quantum and classical physics, quantum physics will appear frequently in this book, in many ways.

The amount and variety of material covered in this book may seem overwhelming. If so, please keep in mind the key goals of the book: to teach the fundamental concepts, which are not so extensive that they should overwhelm, and to illustrate those concepts. Our goal is *not* to provide a *mastery* of the many illustrative applications contained in the book, but rather to convey the spirit of how to apply the basic concepts of classical physics. To help students and readers who feel overwhelmed, we have labeled as "Track Two" sections that can easily be skipped on a first reading, or skipped entirely — but are sufficiently interesting that many readers may choose to browse or study them. Track-Two sections are labeled by the symbol **T2**. To keep Track One manageable for a one-year course, the Track-One portion of each chapter is no longer than 40 pages (including many pages of exercises) and often somewhat shorter.

This book will also seem much more manageable and less overwhelming when one realizes that the same concepts and problem solving techniques appear over and over again, in a variety of different subjects and applications. *These unifying concepts and techniques are listed in outline form in Appendix B, along with the specific applications and section numbers in this book, where they arise.* The reader may also find Appendix A useful. It contains an outline of the entire book based on concepts — an outline complementary to the Table of Contents.

This book is divided into seven parts; see the Table of Contents:

I. Foundations — which introduces a powerful *geometric* point of view on the laws of physics (a viewpoint that we shall use throughout this book), and brings readers up to speed on some concepts and mathematical tools that we shall need. Many readers will already have mastered most or all of the material in Part I, and may find that they can understand most of the rest of the book without adopting our avowedly geometric viewpoint. Nevertheless, we encourage such readers to browse Part I, at least briefly, before moving onward, so as to become familiar with our viewpoint. It does have great power.

Part I is split into two chapters: Chap. 1 on Newtonian Physics; Chap. 2 on Special Relativity. Since the vast majority of Parts II–VI is Newtonian, readers may choose to skip Chap. 2 and the occasional special relativity sections of subsequent chapters, until they are ready to launch into Part VII, General Relativity. Accordingly Chap. 2 is labeled Track Two, though it becomes Track One when readers embark on Part VII.

II. Statistical physics — including kinetic theory, statistical mechanics, statistical thermodynamcs, and the theory of random processes. These subjects underly some portions of the rest of the book, especially plasma physics and fluid mechanics. Among the applications we study are the statistical-theory computation of macroscopic properties of matter (equations of state, thermal and electric conductivity, viscosity, ...); phase transitions (boiling and condensation, melting and freezing, ...); the Ising model and renormalization group; chemical and nuclear reactions, e.g. in nuclear reactors; Bose-Einstein condensates; Olber's Paradox in cosmology; the Greenhouse effect and

its influence on the earth's climate; noise and signal processing, the relationship between information and entropy; entropy in the expanding universe; and the entropy of black holes.

III. Optics — by which we mean classical waves of all sorts: light waves, radio waves, sound waves, water waves, waves in plasmas, and gravitational waves. The major concepts we develop for dealing with all these waves include geometric optics, diffraction, interference, and nonlinear wave-wave mixing. Some of the applications we will meet are gravitational lenses, caustics and catastrophes, Berry's phase, phase-contrast microscopy, Fourier-transform spectroscopy, radio-telescope interferometry, gravitational-wave interferometers, holography, frequency doubling and phase conjugation in nonlinear crystals, squeezed light, and how information is encoded on BD's, DVD's and CD's.

IV. Elasticity — elastic deformations, both static and dynamic, of solids. Here some of our applications are bifurcations of equilibria and bifurcation-triggered instabilities, stress-polishing of mirrors, mountain folding, buckling, seismology and seismic tomography, and elasticity of DNA molecules.

V. Fluid Dynamics — with the fluids including, for example, air, water, blood, and interplanetary and interstellar gas. Among the fluid concepts we study are vorticity, turbulence, boundary layers, subsonic and supersonic flows, convection, sound waves, shock waves and magnetohydrodynamics. Among our applications are the flow of blood through constricted vessels, the dynamics of a high-speed spinning baseball, how living things propel themselves, convection in stars, helioseismology, supernovae, nuclear explosions, sedimentation and nuclear winter, the excitation of ocean waves by wind, salt fingers in the ocean, tornados and water spouts, the Sargasso Sea and the Gulf Stream in the Atlantic Ocean, nonlinear waves in fluids (solitons and their interactions), stellerators, tokamaks, and controlled thermonuclear fusion.

VI. Plasma Physics — with the plasmas including those in earth-bound laboratories and technological devices, the earth's ionosphere, stellar interiors and coronae, and interplanetary and interstellar space. In addition to magnetohydrodynamics (treated in Part V), we develop three other physical and mathematical descriptions of plasmas: kinetic theory, two-fluid formalism, and quasi-linear theory which we express in the quantum language of weakly coupled plasmons and particles. Among our plasma applications are: some of the many types of waves (plasmons) that a plasma can support—both linear waves and nonlinear (soliton) waves; the influence of the earth's ionosphere on radio-wave propagation; the wide range of plasma instabilities that have plagued the development of controlled thermonuclear fusion; and wave-particle (plasmon-electron and plasmon-ion) interactions, including the two-stream instability for fast coronal electrons in the solar wind, isotropization of cosmic rays via scattering by magnetosonic waves, and Landau damping of electrostatic waves.

VII. General Relativity — the physics of curved spacetime, including the laws by which mass-energy and momentum curve spacetime, and by which that curvature influences

the motion of matter and inflluences the classical laws of physics (e.g., the laws of fluid mechanics, electromagntic fields, and optics). Here our applications include, among others, gravitational experiments on earth and in our solar system; relativistic stars and black holes, both spinning (Kerr) and nonspinning (Schwarzschild); the extraction of spin energy from black holes; interactions of black holes with surrounding and infalling matter; gravitational waves and their generation and detection; and the large-scale structure and evolution of the universe (cosmology), including the big bang, the inflationary era, and the modern era. Throughout, we emphasize the physical content of general relativity and the connection of the theory to experiment and observation.

This book's seven Parts are semi-independent of each other. It should be possible to read and teach the parts independently, if one is willing to dip into earlier parts occasionally, as needed, to pick up an occasional concept, tool or result. We have tried to provide enough cross references to make this possible.

Track One of the book has been designed for a full-year course at the first-year graduate level; and that is how we have used it, covering Part I in the first week, and then on average one chapter per week thereafter. (Many fourth-year undergraduates have taken our course successfully, but not easily.)

Exercises are a major component of this book. There are five types of exercises:

1. *Practice.* Exercises that give practice at mathematical manipulations (e.g., of tensors).

2. *Derivation.* Exercises that fill in details of arguments or derivations which are skipped over in the text.

3. *Example.* Exercises that lead the reader step by step through the details of some important extension or application of the material in the text.

4. *Problem.* Exercises with few if any hints, in which the task of figuring out how to set the calculation up and get started on it often is as difficult as doing the calculation itself.

5. *Challenge.* An especially difficult exercise whose solution may require that one read other books or articles as a foundation for getting started.

We urge readers to try working many of the exercises, and *read and think about all of the Example exercises.* The Examples should be regarded as continuations of the text; they contain many of the most illuminating applications. We label with double stars, **, Example exercises that are especially important.

A few words on **units:** In this text we will be dealing with practical matters and will frequently need to have a quantitative understanding of the magnitudes of various physical quantities. This requires us to adopt a particular unit system. Students we teach are about equally divided in preferring cgs/Gaussian units or MKS/SI units. Both of these systems provide a complete and internally consistent set for all of physics and it is an often-debated issue as to which is the more convenient or aesthetically appealing. We will not enter this debate! One's choice of units should not matter and a mature physicist should be able to

change from one system to another with only a little thought. However, when learning new concepts, having to figure out "where the 4π's go" is a genuine impediment to progress. Our solution to this problem is as follows: We shall use the units that seem most natural for the topic at hand or those which, we judge, constitute the majority usage for the subculture that the topic represents. We shall not pedantically convert cm to m or *vice versa* at every juncture; we trust that the reader can easily make whatever translation is necessary. However, where the equations are actually different, for example in electromagnetic theory, we shall sometimes provide, in brackets or footnotes, the equivalent equations in the other unit system and enough information for the reader to proceed in his or her preferred scheme. As an aid, we also give some unit-conversion information in Appendix C, and values of physical constants in Appendix D.

We have written this book in connection with a full-year course that we and others have taught at Caltech nearly every year since the early 1980s. We conceived that course and this book in response to a general concern at Caltech that our PhD physics students were being trained too narrowly, without exposure to the basic concepts of classical physics beyond electricity and magnetism, classical mechanics, and elementary thermodynamics. Courses based on parts of this book, in its preliminary form, have been taught by various physicists, not only at Caltech but also at a few other institutions in recent years, and since moving to Stanford in 2003, Blandford has taught from it there. Many students who took our Caltech course, based on early versions of our book, have told us with enthusiasm how valuable it was in their later careers. Some were even enthusiastic during the course.

Many generations of students and many colleagues have helped us hone the book's presentation and its exercises through comments and criticisms, sometimes caustic, usually helpful; we thank them. Most especially:

For helpful advice about presentations and/or exercises in the book, and/or material that went into the book, we thank Professors Richard Blade, Yanbei Chen, Michael Cross, Steven Frautschi, Peter Goldreich, Steve Koonin, Sterl Phinney, David Politzer, and David Stevenson at Caltech (all of whom taught portions of our Caltech course at one time or another), and XXXXX [ROGER: WHO ELSE SHOULD WE BE LISTING?]

Over the years, we have received extremely valuable advice about this book from the teaching assistants in our course: XXXXXXX[KIP IS ASSEMBLING A LIST]XXXXXXXX We are very indebted to them.

We hope that this book will trigger a significant broadening of the training of physics graduate students elsewhere in the world, as it has done at Caltech, and will be of wide use to mature physicists as well.

Roger D. Blandford and Kip S. Thorne
Stanford University and Caltech, December 2012

CONTENTS

[For an alternative overview of this book, See *Appendix A. Concept-Based Outline* (does not exist yet)]

Preface

I. FOUNDATIONS

1. Newtonian Physics: Geometric Viewpoint

1.1 Introduction

1.2 Foundational Concepts

1.3 Tensor Algebra Without a Coordinate System

Box: [T2] Vectors and tensors in quantum theory

1.4 Particle Kinetics and Lorentz Force in Geometric Language

1.5 Component Representation of Tensor Algebra

1.6 Orthogonal Transformations of Bases

1.7 Directional Derivatives, Gradients, Levi-Civita Tensor, Cross Product and Curl

**Examples: Rotation in x, y Plane; Vector identities for cross product and curl

1.8 Volumes, Integration, and Conservation Laws

1.9 The Stress Tensor and Conservation of Momentum

**Examples: Equations of motion for a perfect fluid; Electromagnetic stress tensor

1.10 Geometrized Units and Relativistic Particles for Newtonian Readers

2. [T2] Special Relativity: Geometric Viewpoint

2.1 Overview

2.2 Foundational Concepts

Boxes: Measuring the speed of light without light; Propagation speeds of other waves; Proof of invariance of the interval for timelike separations

2.3 Tensor Algebra Without a Coordinate System

2.4 Particle Kinetics and Lorentz Force Without a Reference Frame

**Examples: Frame-independent expressions for energy, momentum and velocity; 3-metric as a projection tensor; Doppler shift derived without Lorentz transformations

2.5 Component Representation of Tensor Algebra

2.6 Particle Kinetics in Index Notation and in a Lorentz Frame

2.7 Lorentz Transformations

**Exercise: General boosts and rotations

2.8 Spacetime Diagrams for Boosts

2.9 Time Travel

2.10 Directional Derivatives, Gradients, Levi-Civita Tensor

2.11 Nature of Electric and Magnetic Fields; Maxwell's Equations

2.12 Volumes, Integration and Conservation Laws

2.13 The Stress-Energy Tensor and Conservation of 4-Momentum

**Example: Stress-energy tensor and energy-momentum conservation for a perfect fluid and for the electromagnetic field; Inertial mass per unit volume;

II. STATISTICAL PHYSICS

3. Kinetic Theory

3.1 Overview

3.2 Phase Space and Distribution Function: number density in phase space; distribution function for particles in a plasma; distribution function for photons; mean occupation number

**Examples: [T2] Distribution function for particles with a range of rest masses; Regimes of particulate and wave-like behavior—X-rays from Cygnus X-1 and gravitational waves from a supernova

3.3 Thermal Equilibrium Distribution Functions

**Examples: Maxwell velocity distribution; [T2] Observations of cosmic microwave radiation from earth

3.4 Macroscopic Properties of Matter as Integrals Over Momentum Space: Newtonian particle density, flux and stress tensor; relativistic number-flux 4-vector and stress-energy tensor

3.5 Isotropic Distribution Functions and Equations of State: density, pressure, energy density, equation of state for nonrelativistic hydrogen gas, for relativistic degenerate hydrogren gas, and for radiation

3.6 Evolution of the Distribution Function: Liouville's Theorem, the Collisionless Boltzmann Equation, and the Boltzmann Transport Equation

Box: [T2] Sophisticated derivation of relativistic collisionless Boltzmann equation

**Examples: Solar heating of the earth—the Greenhouse effect; Olber's paradox and solar furnace

3.7 Transport Coefficients: diffusive heat conduction inside a star, analyzed in order of magnitude and via the Boltzmann transport equation

Box: Two lengthscale expansions

**Examples: Solution of diffusion equation in an infinte, homogeneous medium; Diffusion equation for temperature; Viscosity of a monatomic gas; Neutron diffusion in a nuclear reactor

4. Statistical Mechanics

4.1 Overview

4.2 Systems, Ensembles, and Distribution Functions

Box: [T2] Density operator and quantum statistical mechanics

4.3 Liouville's Theorem and the Evolution of the Distribution Function

4.4 Statistical Equilibrium: canonical ensemble and distribution; general ensemble; Gibbs ensemble; grand canonical ensemble; Bose-Einstein and Fermi-Dirac distributions; equipartition theorem

4.5 The Microcanonical Ensemble

4.6 The Ergodic Hypothesis

4.7 Entropy and the Evolution into Statistical Equilibrium: the second law of thermodynamics; what causes entropy to increase?

Box: [T2] Entropy increase due to discarding quantum correlations

**Exercises: Entropy of a thermalized mode of a field; Entropy of mixing, indistinguishability of atoms, and the Gibbs paradox

4.8 Entropy Per Particle

**Exercise: Primordial element formation

4.9 Bose-Einstein Condensate

**Exercise: Onset of Bose-Einstein condensation; Discontinuous change of specific heat

4.10 [T2] Statistical Mechanics in the Presence of Gravity: Galaxies, Black Holes, the Universe, and Structure Formation in the Early Universe

4.11 [T2] Entropy and Information: information gained in measurements; information in communication theory; examples of information content; some properties of information; capacity of communication channels; erasing information from computer memories

5. Statistical Thermodynamics

5.1 Overview

5.2 Microcanonical Ensemble and the Energy Representation of Thermodynamics

5.3 Grand Canonical Ensemble and the Grand Potential Representation of Thermodynamics: computation of thermodynamic properties; van der Waals gas

Box: [T2] Derivation of van der Waals grand potential

5.4 Canonical Ensemble and the Physical Free-Energy Representation of Thermodynamic; ideal gas with internal degrees of freedom

5.5 The Gibbs Representation of Thermodynamics; Phase Transitions and Chemical Reactions

**Exercises: Electron-positron equilibrium at low temperatures; Saha equation for ionization equilibrium

5.6 Fluctuations away from Satistical Equilibrium

5.7 Van der Waals Gas: Volume Fluctuations and Gas-To-Liquid Phase Transition

**Exercise: Out-of-equilibrium Gibbs potential for water; surface tension and nucleation

5.8 [T2] Magnetic materials: Paramagnetism, Ising Model for Ferromagnetism, Renormalization Group, and Monte Carlo Methods

6. Random Processes

6.1 Overview

6.2 Fundamental Concepts: random variables and processes, probability distributions, ergodic hypothesis

6.3 Markov Processes and Gaussian Processes; central limit theorem; random walk; Doob's theorem

**Exercises: Diffusion of a particle; Random walks

6.4 Correlation Functions and Spectral Densities; the Wiener-Khintchine theorem; light spectra; noise in a gravitational-wave detector

**Exercise: Cosmological density fluctuations

6.5 [T2] Two-Dimensional Random Processes

6.6 Noise and its Types of Spectra; Noise in atomic Clocks; Information missing from the spectral density

6.7 Filtering Random Processes; Brownian motion and random walks; Extracting a weak Signal from noise; Band-pass filter; Signal to noise ratio; Shot noise

**Exercises: Wiener's optimal filter; Allan variance for clock noise

6.8 Fluctuation-Dissipation Theorem; Langevin equation; Johnson noise in a resistor; Relaxation time for Brownian motion; [T2] Generalized fluctuation dissipation theorem

**Exercises: Detectability of a sinusoidal force acting on an oscillator with thermal noise; [T2] Standard quantum limit for minimum noise in a linear measuring device, and how to evade it

6.9 Fokker-Planck Equation; Optical molasses (doppler cooling of atoms)

**Exercise: [T2] Solution of Fokker-Planck equation for thermal noise in an oscillator

III. OPTICS

7. Geometrical Optics

7.1 Overview

7.2 Waves in a Homogeneous Medium: monochromatic plane waves; dispersion relation; wave packets; group and phase velocities

Applications to: (i) EM waves in isotropic, dielectric medium, (ii) sound waves in a solid or fluid, (iii) waves on the surface of a deep body of water, (iv) flexural waves on a stiff beam or rod, (v) Alfvén waves in a magnetized plasma

**Exercise: Gaussian wave packet and its dispersion

7.3 Waves in an Inhomogeneous, Time-Varying Medium: The Eikonal approximation; geometric optics; relation to quantum theory; relation to wavepackets; breakdown of geometric optics; Fermat's principle

Box: Bookkeeping parameter in two-lengthscale expansions

**Exercises for dispersionless waves: Amplitude propagation; energy density, energy flux, and adiabatic invariant

**Exercise: Geometric optics for Schrödinger equation; Hamilton-Jacobi theory

Applications: sound waves in wind; spherical scalar waves; flexural waves; Alfven waves; light through a lens; self-focusing optical fibers

7.4 Paraxial Optics

Applications: lenses and mirrors, telescope, microscope, optical cavity, converging magnetic lens for charged particle beam

7.5 Catastrophe Optics: Multiple Images; Formation of Caustics and their Properties

Applications: Imperfect lens, sunlight's caustic patterns on the bottom of a swimming pool and through a water glass

**Exercises: Catastrophe theory – the five elementary catastrophes applied to optical caustics and to the van der Waals phase change

7.6 [T2] Gravitational Lenses: Refractive index model; Multiple Images and Caustics

[T2] Applications: Lensing by a point mass; lensing of a quasar by a galaxy

7.7 Polarization: its geometric-optics propagation

[T2] Application: The Geometric Phase

8. Diffraction

8.1 Overview

8.2 Helmholtz-Kirchhoff Integral: diffraction by an aperture; spreading of the wavefront

8.3 Fraunhofer Diffraction: diffraction grating; Airy pattern; Hubble space telescope; Babinet's principle

Other Applications: Pointilist paintings; light scattering by large opaque particle; measuring thickness of human hair via diffraction

8.4 Fresnel Diffraction: Fresnel integrals and Cornu spiral; lunar occultation of a radio source; circular apertures, Fresnel zones and zone plates

Other Applications: seeing (stellar scintillation) in the atmosphere; multiconjugate adaptive optics; spy satellites

8.5 Paraxial Fourier Optics: coherent illumination; point spread functions; Abbé theory of image formation by a thin lens; spatial filtering in the focal plane of a lens; Gaussian beams; gravitational-wave interferometers

Other applications of paraxial Fourier optics — high-pass filter to clean a laser beam; low-pass filter to enhance sharp features in an image; notch filter to remove pixellations from an image; phase contrast microscopy; spatial pattern recognition; convolution; Gaussian beams in an optical fiber; scattered-light noise in LIGO

**Example: Transmission electron microscope

8.6 Diffraction at Caustics; scaling laws

9. Interference

9.1 Overview

9.2 Coherence: Young's slits; extended source; van Cittert-Zernike theorem; spatial lateral coherence; 2-dimensional coherence; Michelson stellar interferometer and atmospheric seeing; temporal coherence; Michelson interferometer; Fourier transform spectroscopy; degree of coherence

**Example: Complex random processes and van Cittert-Zernike theorem

9.3 Radio Telescopes: two-element interferometer; multiple element interferometer; closure phase; angular resolution

9.4 Etalons and Fabry-Perot Interferometers: multiple-beam interferometry; modes of a Fabry-Perot cavity

Fabry-Perot applications: spectrometer, laser, mode-cleaning cavity, beam-shaping cavity, PDH laser stabilization, optical frequency comb

**Examples: transmission and reflection coefficients; reciprocity relations; antireflection coating; Sagnac interferometer

9.5 [T2] Laser Interferometer Gravitational Wave Detectors

9.6 Power Correlations and Photon Statistics: Hanbury Brown & Twiss Intensity Interferometer

10. Nonlinear Optics

10.1 Overview

10.2 Lasers: Basic Principles; Types of Lasers their performances, and applications; details of Ti:Sapp mode-locked laser

10.3 Holography: recording a hologram and reconstructing a 3D image from it

Other types of holography and applications: phase holography, volume holography, reflection holography, white-light holography, computational holograms, full-color holography, holographic interferometry, holographic lenses

**Problem: CDs, DVDs and Blu Ray disks

10.4 Phase-Conjugate Optics

10.5 Maxwell's Equations in a Nonlinear Medium: nonlinear dielectric susceptibilities; electro-optic effects

Box: properties of some anisotropic, nonlinear crystals

10.6 Three-Wave Mixing in Anisotropic, Nonlinear Crystals: resonance conditions; evolution equations in a medium that is isotropic at linear order; three-wave mixing in a birefringent crystal: phase matching and evolution equations

10.7 Applications of Three-Wave Mixing: Frequency doubling; optical parametric amplification; squeezed light

**Example: frequency doubling in a green laser pointer; qqueezing by children who swing; squeezed states of light

10.8 Four-wave mixing in isotropic media: third-order susceptibilities and field strengths; phase conjugation via four-wave mixing in Cs_2; optical Kerr effect and four-wave mixing in an optical fiber.

Table: materials used in four-wave mixing; **Problems: spontaneous oscillation in four-wave mixing; squeezed light produced by phase conjugation

IV. ELASTICITY

11. Elastostatics

11.1 Overview

11.2 Displacement and Strain; expansion, rotation, and shear

11.3 Stress and Elastic Moduli: stress tensor; elastostatic stress balance; energy of deformation; molecular origin of elastic stress

11.4 Young's Modulus and Poisson Ratio for an Isotropic Material

11.5 [T2] Cylindrical and Spherical Coordinates: connection coefficients and components of strain

11.6 [T2] Solving the 3-Dimensional Elastostatic Equations in Cylindrical Coordinates: simple methods—pipe fracture and torsion pendulum; separation of variables and Green's functions—thermoelastic noise in a LIGO mirror

11.7 Reducing the Elastostatic Equations to One Dimension for a Bent Beam: cantilever bridges; elastica

11.8 Bifurcation of Equilibria; Buckling and Mountain Folding

11.9 [T2] Reducing the Elastostatic Equations to Two Dimensions for a Deformed This Plate: stress-polishing a telescope mirror

12. Elastodynamics

12.1 Overview

12.2 Conservation Laws

12.3 Basic Equations of Elastodynamics: equation of motion; elastodynamic waves; longitudinal sound waves; transverse shear waves; energy of elastodynamic waves

12.4 Waves in Rods, Strings and Beams: compression waves; torsion waves; waves on strings; flexural waves on a beam; bifurcation of equilibria and buckling (once more)

12.5 Body and Surface Waves — Seismology: body waves; edge waves; Green's function for a homogeneous half space; free oscillations of solid bodies; seismic tomography

12.6 The Relationship of Classical Waves to Quantum Mechanical Excitations

V. FLUID DYNAMICS

13. Foundations of Fluid Dynamics

13.1 Overview

13.2 The Macroscopic Nature of a Fluid: Density, Pressure, Flow Velocity; Fluids vs. gases

13.3 Hydrostatics: Archimedes law; stars and planets; rotating fluids

13.4 Conservation Laws

13.5 The Dynamics of an Ideal Fluid: mass conservation; momentum conservation; Euler equation; Bernoulli theorem; conservation of energy; Joule-Kelvin cooling

13.6 Incompressible Flows

13.7 Viscous Flows with Heat Conduction: decomposition of the velocity gradient into expansion, vorticity, and shear; Navier-Stokes equation; energy conservation and entropy production; molecular origin of viscosity; Reynolds' number; pipe flow

13.8 [T2] Relativistic Dynamics of an Ideal Fluid: stress-energy tensor and equations of relativistic fluid mechanics; relativistic Bernoulli equation and ultrarelativistic astrophysical jets; nonrelativistic limt

14. Vorticity

14.1 Overview

14.2 Vorticity and Circulation: vorticity transport; vortex lines; tornados; Kelvin's theorem; diffusion of vortex lines; sources of vorticity

14.3 Low Reynolds' Number Flow — Stokes' flow: sedimentation; nuclear winter

14.4 High Reynolds' Number Flow — Laminar Boundary Layers: similarity solution; vorticity profile; separation

14.5 Nearly Rigidly Rotating Flow — Earth's Atmosphere and Oceans: fluid dynamics in a rotating reference frame; geostrophic flows; Taylor-Proudman theorem; Ekman boundary layers

14.6 [T2] Instabilities of Shear Flows — Billow Clouds, Turbulence in the Stratosphere: discontinuous flow, Kelvin-Helmholtz instability; discontinuous flow with gravity; smoothly stratified flows, Rayleigh and Richardson criteria for instability

15. Turbulence

15.1 Overview

15.2 The Transition to Turbulence — Flow past a Cylinder

15.3 Empirical Description of Turbulence: the role of vorticity in turbulence

15.4 Semi-Quantitative Analysis of Turbulence: weak turbulence; turbulent viscosity; turbulent wakes and jets; entrainment and the Coanda effect; Kolmogorov spectrum

15.5 Turbulent Boundary Layers: profile of a turbulent boundary layer; instability of a laminar boundary layer; the flight of a ball

15.6 The Route to Turbulence — Onset of Chaos: Couette flow; Feigenbaum sequence and onset of turbulence in convection

16. Waves

16.1 Overview

16.2 Gravity Waves on Surface of a Fluid: deep water waves; shallow water waves; surface tension; capillary waves; tsunamis; helioseismology

16.3 Nonlinear Shallow Water Waves and Solitons: Korteweg-deVries equation; physical effects in the kdV equation; single soliton solution; two soliton solution; solitons in contemporary physics

16.4 Rossby Waves in a Rotating Fluid

16.5 Sound Waves: wave energy; sound generation; [T2] radiation reaction, runaway solutions and matched asymptotic expansions

17. Compressible and Supersonic Flow

17.1 Overview

17.2 Equations of Compressible Flow

17.3 Stationary, Irrotational Flow: quasi-one-dimensional flow; setting up a stationary transonic flow; rocket engines

17.4 One Dimensional, Time-Dependent Flow: Riemann invariants; shock tube

17.5 Shock Fronts: shock jump conditions; Rankine-Hugoniot relations; internal structure of a shock; jump conditions in perfect gas with constant γ; Mach cone

17.6 Self-Similar Solutions — Sedov-Taylor Blast Wave: atomic bomb; supernovae

18. Convection

18.1 Overview

18.2 [T2] Diffusive Heat Conduction: cooling a nuclear reactor; thermal boundary layers

18.3 [T2] Boussinesq Approximation

18.4 [T2] Rayleigh-Bénard Convection: mantle convection and continental drift

18.5 Convection in Stars

18.6 [T2] Double Diffusion: salt fingers

19. Magnetohydrodynamics

19.1 Overview

19.2 Basic Equations of MHD: Maxwell's equations in MHD approximation; momentum and energy conservation; boundary conditions; magnetic field and vorticity

19.3 Magnetostatic Equilibria: controlled thermonuclear fusion; Z pinch; θ pinch; tokamak

19.4 Hydromagnetic Flows: electromagnetic brake; MHD power generator; flow meter; electromagnetic pump; Hartmann flow

19.5 Stability of Hydromagnetic Equilibria: linear perturbation theory; Z pinch – sausage and kink instabilities; energy principle

19.6 Dynamos and Magnetic Field Line Reconnection: Cowling's theorem; kinematic dynamos; magnetic reconnection

19.7 Magnetosonic Waves and the Scattering of Cosmic Rays

VI. PLASMA PHYSICS

20. The Particle Kinetics of Plasmas

20.1 Overview

20.2 Examples of Plasmas and their Density-Temperature Regimes: ionization boundary; degeneracy boundary; relativistic boundary; pair production boundary; examples of natural and man-made plasmas

20.3 Collective Effects in Plasmas: Debye shielding; collective behavior; plasma oscillations and plasma frequency

20.4 Coulomb Collisions: collision frequency; Coulomb logarithm; thermal equilibration times

20.5 Transport Coefficients: anomalous resistivity and anomalous equilibration

20.6 Magnetic field: Cyclotron frequency and Larmor radius; validity of the fluid approximation; conductivity tensor

20.7 Adiabatic invariants: homogeneous time-independent electric and magnetic fields; inhomogeneous time-independent magnetic field; a slowly time-varying magnetic field

21. Waves in Cold Plasmas: Two-Fluid Formalism

21.1 Overview

21.2 Dielectric Tensor, Wave Equation, and General Dispersion Relation

21.3 Two-Fluid Formalism

21.4 Wave Modes in an Unmagnetized Plasma: dielectric tensor and dispersion relation for a cold plasma; electromagnetic plasma waves; Langmuir waves and ion acoustic waves in a warm plasma; cutoffs and resonances

21.5 Wave Modes in a Cold, Magnetized Plasma: dielectric tensor and dispersion relation; parallel propagation; perpendicular propagation

21.6 Propagation of Radio Waves in the Ionosphere

21.7 CMA Diagram for Wave Modes in Cold, Magnetized Plasma

21.8 Two-Stream Instability

22. Kinetic Theory of Warm Plasmas

22.1 Overview

22.2 Basic Concepts of Kinetic Theory and its Relationship to Two-Fluid Theory: distribution function and Vlasov equation; Jeans' theorem

22.3 Electrostatic Waves in an Unmagnetized Plasma and Landau Damping: formal dispersion relation; two-stream instability; the Landau contour; dispersion relation for weakly damped or growing waves; Langmuir waves and their Landau damping; ion acoustic waves and conditions for their Landau damping to be weak

22.4 Stability of Electromagnetic Waves in an Unmagnetized Plasma

22.5 Particle Trapping

22.6 [T2] N-Particle Distribution Function: BBKGY hierarchy, two-point correlation function, Coulomb correction to plasma pressure

23. Nonlinear Dynamics of Plasmas

23.1 Overview

23.2 Quasi-Linear Theory in Classical Language: classical derivation of the theory; summary of the theory; conservation laws; generalization to three dimensions

23.3 Quasilinear Theory in Quantum Mechanical Language: plasmon occupation number η; evolution of plasmons via interaction with electrons; evolution electrons via interaction with plasmons; emission of plasmons by particles in presence of a magnetic field; relationship between classical and quantum formalisms; three-wave moxing

23.4 Quasilinear Evolution of Unstable Distribution Function — The Bump in Tail: instability of streaming cosmic rays

23.5 Parametric Instabilities

23.6 Solitons and Collisionless Shock Waves

VII. GENERAL RELATIVITY

24. From Special to General Relativity

24.1 Overview

24.2 Special Relativity Once Again: geometric, frame-independent formulation; inertial frames and components of vectors, tensors and physical laws; light speed, the interval, and spacetime diagrams

24.3 Differential Geometry in General Bases and in Curved Manifolds: nonorthonormal bases; vectors as differential operators; tangent space; commutators; differentiation of vectors and tensors; connection coefficients; integration

24.4 Stress-Energy Tensor Revisited

24.5 Proper Reference Frame of an Accelerated Observer: relation to inertial coordinates; metric in proper reference frame; transport law for rotating vectors; geodesic equation for freely falling particle; uniformly accelerated observer; Rindler coordinates for Minkowski spacetime

25. Fundamental Concepts of General Relativity

25.1 Overview

25.2 Local Lorentz Frames, the Principle of Relativity, and Einstein's Equivalence Principle

25.3 The Spacetime Metric, and Gravity as a Curvature of Spacetime

25.4 Free-fall Motion and Geodesics of Spacetime

25.5 Relative Acceleration, Tidal Gravity, and Spacetime Curvature: Newtonian description of tidal gravity; relativistic description; comparison of descriptions

25.6 Properties of the Riemann curvature tensor

25.7 Curvature Coupling Delicacies in the Equivalence Principle, and some Non-gravitational Laws of Physics in Curved Spacetime

25.8 The Einstein Field Equation

25.9 Weak Gravitational Fields: Newtonian limit of general relativity; linearized theory; gravitational field outside a stationary, linearized source; conservation laws for mass, momentum and angular momentum; tidal and frame-drag fields

26. Relativistic Stars and Black Holes

26.1 Overview

26.2 Schwarzschild's Spacetime Geometry

26.3 Static Stars: Birkhoff's theorem; stellar interior; local energy and momentum conservation; Einstein field equation; stellar models and their properties; embedding diagrams

26.4 Gravitational Implosion of a Star to Form a Black Hole: tidal forces at the gravitational radius; stellar implosion in Eddington-Finkelstein coordinates; tidal forces at $r = 0$ — the central singularity; Schwarschild black hole

26.5 Spinning Black Holes: the Kerr metric for a spinning black hole; dragging of inertial frames; light-cone structure and the horizon; evolution of black holes — rotational energy and its extraction; [T2] tendex and vortex lines

26.6 The Many-Fingered Nature of Time

27. Gravitational Waves and Experimental Tests of General Relativity

27.1 Overview

27.2 Experimental Tests of General Relativity: equivalence principle, gravitational redshift, and global positioning system; perihelion advance of Mercury; gravitational deflection of light, Fermat's principle and gravitational lenses; Shapiro time delay; frame dragging and Gravity Probe B; binary pulsar

27.3 Gravitational Waves Propagating Through Flat Spacetime: weak plane waves in linearized theory; measuring a gravitational wave by its tidal forces; tendex and vortex lines for a gravitational wave; gravitons and their spin and rest mass

27.4 Gravitational Waves Propagating Through Curved Spacetime: gravitational wave equation in curved spacetime; geometric-optics propagation of gravitational waves; energy and momentum in gravitational waves

27.5 The Generation of Gravitational Waves: multipole-moment expansion; quadrupole moment formalism; quadrupolar wave strength, energy, angular momentum and radiation reaction; gravitational waves from a binary star system; [T2] gravitational waves from binaries made of black holes and/or neutron stars — numerical relativity

27.6 The Detection of Gravitational Waves: frequency bands and detection techniques; gravitational-wave interfereomters: overview and elementary treatment; [T2] interferometer analyzed in TT gauge; [T2] interferometer analyzed in proper reference frame of beam splitter; [T2] realistic interferometers

28. Cosmology

28.1 Overview

28.2 Homogeneity and Isotropy of the Universe — Robertson-Walker line element

28.3 The Stress-energy Tensor and the Einstein Field Equation

28.4 Evolution of the Universe: constituents of the universe — cold matter, radiation, and dark energy; the vacuum stress-energy tensor; evolution of the densities; evolution in time and redshift; physical processes in the expanding universe

28.5 Observational Cosmology: parameters characterizing the universe; local Lorentz frame of homogenous observers near Earth; Hubble expansion rate; primordial nucleosynthesis; density of cold dark matter; radiation temperature and density; anisotropy of the CMB: measurements of the Doppler peaks; age of the universe — constraint on the dark energy; magnitude-redshift relation for type Ia supernovae — confirmation that the universe is accelerating

28.6 The Big-Bang Singularity, Quantum Gravity and the Intial Conditions of the Universe

28.7 Inflationary Cosmology: amplification of primordial gravitational waves by inflation; search for primordial gravitational waves by their influence on the CMB; probing the inflationary expansion rate

APPENDICES

Appendix A: Concept-Based Outline of this Book

Appendix B: Unifying Concepts

Appendix C: Units

Appendix D: Values of Physical Constants

Another widely known physics application of the variational method is Fermatâ€™s Principle in ray optics. It states that light rays between two xed points A and B traverse the path of sta-tionary optical length with respect to variations of the path, i.e., An important consideration in the construction of our. classical-physics Finsler examples concerns the dimensional-. ity of the manifold involved. For example, the aforementioned.