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Algebraic independence of periods of elliptic curves

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References

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Some of these paper are available on the internet. See in particular

http://www.imj-prg.fr/~michel.waldschmidt/texts.html

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Several subsequent papers analyzed the performance of other forms of elliptic curves proposed in the mathematical literature. See, e.g., [18] for the speed of several dialects of the Weierstrass form, [34] for the speed of Jacobi intersections, [28] for the speed of Hessians, and [9] for the speed of Jacobi quartics; see also [38] and [23], which introduced the Montgomery and Doche/Icart/Kohel forms and analyzed their speed. An elliptic curve over $k$ is a nonsingular projective algebraic curve $E$ of genus 1 over $k$ with a chosen base point $O \in E$. Remark. There is a somewhat subtle point here concerning what is meant by a point of a curve over a non-algebraically-closed field. Any elliptic curve $E$ over $k$ is isomorphic to the curve in $\mathbb{P}^2_k$ dened by some generalised Weierstrass equation, with the base point $O$ of $E$ being mapped to $(0 : 1 : 0)$. Conversely any non-singular generalised Weierstrass equation denes an elliptic curve, with this choice of basepoint. Proposition 1.6, called periods of $E$ and let $\Lambda = \mathbb{Z}\omega_1 + \mathbb{Z}\omega_2$. It can be shown that $\omega_1, \omega_2$ are linearly independent over $\mathbb{R}$ and hence $\Lambda$ is a lattice. Fix a point $P_0 \in E(\mathbb{C})$ and define the map $\psi : E(\mathbb{C}) \rightarrow \mathbb{C}/\Lambda$. For further accounts of these types of results and their history, I highly recommend Waldschmidt's articles "Transcendence of periods: the state of the art," Pure Appl. Math. Q. 2 (2006), no. 2, part 2, 435-463, and "Elliptic functions and transcendence," Surveys in number theory, Dev.