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A Chronology of Game Theory

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| Ancient | 1700 | 1800 | 1900 | 1950 | 1960 | 1970 | 1980 | 1990 | Nobel Prize | 2nd Nobel Prize | 3rd Nobel Prize |

0-500AD

The Babylonian Talmud is the compilation of ancient law and tradition set down during the first five centuries A.D. which serves as the basis of Jewish religious, criminal and civil law. One problem discussed in the Talmud is the so called marriage contract problem: a man has three wives whose marriage contracts specify that in the case of this death they receive 100, 200 and 300 respectively. The Talmud gives apparently contradictory recommendations. Where the man dies leaving an estate of only 100, the Talmud recommends equal division. However, if the estate is worth 300 it recommends proportional division (50,100,150), while for an estate of 200, its recommendation of (50,75,75) is a complete mystery. This particular Mishna has baffled Talmudic scholars for two millennia. In 1985, it was recognised that the Talmud anticipates the modern theory of cooperative games. Each solution corresponds to the nucleolus of an appropriately defined game.

1713

In a letter dated 13 November 1713 Francis Waldegrave provided the first, known, minimax mixed strategy solution to a two-person game. Waldegrave wrote the letter, about a two-person version of the card game le Her, to Pierre-Remond de Montmort who in turn wrote to Nicolas Bernoulli, including in his letter a discussion of the Waldegrave solution. Waldegrave's solution is a minimax mixed strategy equilibrium, but he made no extension of his result to other games, and expressed concern that a mixed strategy "does not seem to be in the usual rules of play" of games of chance

1838

Publication of Augustin Cournot's Researches into the Mathematical Principles of the Theory of Wealth. In chapter 7, On the Competition of Producers, Cournot discusses the special case of duopoly and utilises a solution concept that is a restricted version of

the Nash equilibrium

1871

In the first edition of his book The Descent of Man, and Selection in Relation to Sex Charles Darwin gives the first (implicitly) game theoretic argument in evolutionary biology. Darwin argued that natural section will act to equalize the sex ratio. If, for example, births of females are less common than males, then a newborn female will have better mating prospects than a newborn male and therefore can expect to have more offspring. Thus parents genetically disposed to produce females tend to have more than the average numbers of grandchildren and thus the genes for female-producing tendencies spread, and female births become commoner. As the 1:1 sex ratio is approached, the advantage associated with producing females dies away. The same reasoning holds if males are substituted for females throughout. Therefore 1:1 is the equilibrium ratio.

1881

Publication of Francis Ysidro Edgeworth's Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences. Edgeworth proposed the contract curve as a solution to the problem of determining the outcome of trading between individuals. In a world of two commodities and two types of consumers he demonstrated that the contract curve shrinks to the set of competitive equilibria as the number of consumers of each type becomes infinite. The concept of the core is a generalisation of Edgeworth's contract curve.

1913

The first 'theorem' of game theory asserts that in chess either white can force a win, or black can force a win, or both sides can force at least a draw. This 'theorem' was published by Ernst Zermelo in his paper Uber eine Anwendung der Mengenlehre auf die Theorie des Schachspiels and hence is referred to as Zermelo's Theorem. Zermelo's results were extended and generalised in two papers by Denes Konig and Laszlo Kalmar. The Kalmar paper contains the first proof of Zermelo's theorem since Zermelo's own paper did not give one. An English translation of the Zermelo paper, along with a discussion its significance and its relationship to the work of Konig and Kalmar is contained in Zermelo and the Early History of Game Theory by U. Schwalbe and P. Walker.

1921-27

Emile Borel published four notes on strategic games and an erratum to one of them. Borel gave the first modern formulation of a mixed strategy along with finding the minimax solution for two-person games with three or five possible strategies. Initially he maintained that games with more possible strategies would not have minimax solutions, but by 1927, he considered this an open question as he had been unable to find a counterexample.

1928

John von Neumann proved the minimax theorem in his article Zur Theorie der Gesellschaftsspiele. It states that every two- person zero-sum game with finitely many pure strategies for each player is determined, ie: when mixed strategies are admitted, this variety of game has precisely one individually rational payoff vector. The proof makes involved use of some topology and of functional calculus. This paper also

introduced the extensive form of a game.

1930

Publication of F. Zeuthen's book Problems of Monopoly and Economic Warfare. In chapter IV he proposed a solution to the bargaining problem which Harsanyi later showed is equivalent to Nash's bargaining solution.

1934

R.A. Fisher independently discovers Waldegrave's solution to the card game le Her. Fisher reported his work in the paper Randomisation and an Old Enigma of Card Play.

1938

Ville gives the first elementary, but still partially topological, proof of the minimax theorem. Von Neumann and Morgenstern's (1944) proof of the theorem is a revised, and more elementary, version of Ville's proof.

1944

Theory of Games and Economic Behavior by John von Neumann and Oskar Morgenstern is published. As well as expounding two-person zero sum theory this book is the seminal work in areas of game theory such as the notion of a cooperative game, with transferable utility (TU), its coalitional form and its von Neumann-Morgenstern stable sets. It was also the account of axiomatic utility theory given here that led to its wide spread adoption within economics.

1945

Herbert Simon writes the first review of von Neumann-Morgenstern.

1946

The first entirely algebraic proof of the minimax theorem is due to L. H. Loomis's, On a Theorem of von Neumann, paper.

1950

Contributions to the Theory of Games I, H. W. Kuhn and A. W. Tucker eds., published.

1950

In January 1950 Melvin Dresher and Merrill Flood carry out, at the Rand Corporation, the experiment which introduced the game now known as the Prisoner's Dilemma. The famous story associated with this game is due to A. W. Tucker, A Two-Person Dilemma, (memo, Stanford University). Howard Raiffa independently conducted, unpublished, experiments with the Prisoner's Dilemma.

1950

John McDonald's Strategy in Poker, Business and War published. This was the first introduction to game theory for the general reader.

1950-53

Extensive form games allow the modeller to specify the exact order in which players have to make their decisions and to formulate the assumptions about the information possessed by the players in all stages of the game. In two papers, Extensive Games (1950) and Extensive Games and the Problem of Information (1953), H. W. Kuhn included the formulation of extensive form games which is currently used, and also some basic theorems pertaining to this class of games.

1950-53

In four papers between 1950 and 1953 John Nash made seminal contributions to both non-cooperative game theory and to bargaining theory. In two papers, Equilibrium

Points in N- Person Games (1950) and Non-cooperative Games (1951), Nash proved the existence of a strategic equilibrium for non-cooperative games-the Nash equilibrium-and proposed the "Nash program", in which he suggested approaching the study of cooperative games via their reduction to non-cooperative form. In his two papers on bargaining theory, The Bargaining Problem (1950) and Two-Person Cooperative Games (1953), he founded axiomatic bargaining theory, proved the existence of the Nash bargaining solution and provided the first execution of the Nash program.

1951

George W. Brown described and discussed a simple iterative method for approximating solutions of discrete zero-sum games in his paper Iterative Solutions of Games by Fictitious Play.

1952

The first textbook on game theory was John Charles C. McKinsey, Introduction to the Theory of Games.

1952

Merrill Flood's report, (Rand Corporation research memorandum, Some Experimental Games, RM-789, June), on the 1950 Dresher/Flood experiments appears.

1952

The Ford Foundation and the University of Michigan sponsor a seminar on the "Design of Experiments in Decision Processes" in Santa Monica. This was the first experimental economics/experimental game theory conference

1952-53

The notion of the Core as a general solution concept was developed by L. S. Shapley (Rand Corporation research memorandum, Notes on the N-Person Game III: Some Variants of the von-Neumann-Morgenstern Definition of Solution, RM- 817, 1952) and D. B. Gillies (Some Theorems on N-Person Games, Ph.D. thesis, Department of Mathematics, Princeton University, 1953). The core is the set of allocations that cannot be improved upon by any coalition.

1953

Lloyd Shapley in his paper A Value for N-Person Games characterised, by a set of axioms, a solution concept that associates with each coalitional game,v, a unique out-come, v. This solution in now known as the Shapley Value.

1953

Lloyd Shapley's paper Stochastic Games showed that for the strictly competitive case, with future payoff discounted at a fixed rate, such games are determined and that they have optimal strategies that depend only on the game being played, not on the history or even on the date, ie: the strategies are stationary.

1953

Contributions to the Theory of Games II, H. W. Kuhn and A. W. Tucker eds., published.

1954

One of the earliest applications of game theory to political science is L. S. Shapley and M. Shubik with their paper A Method for Evaluating the Distribution of Power in a Committee System. They use the Shapley value to determine the power of the

members of the UN Security Council.

1954-55

Differential Games were developed by Rufus Isaacs in the early 1950s. They grew out of the problem of forming and solving military pursuit games. The first publications in the area were Rand Corporation research memoranda, by Isaacs, RM-1391 (30 November 1954), RM-1399 (30 November 1954), RM-1411 (21 December 1954) and RM-1486 (25 March 1955) all entitled, in part, Differential Games.

1955

One of the first applications of game theory to philosophy is R. B. Braithwaite's Theory of Games as a Tool for the Moral Philosopher.

1957

Games and Decisions: Introduction and Critical Survey by Robert Duncan Luce and Howard Raiffa published.

1957

Contributions to the Theory of Games III, M. A. Dresher, A. W. Tucker and P. Wolfe eds., published.

1959

The notion of a Strong Equilibrium was introduced by R. J. Aumann in the paper Acceptable Points in General Cooperative N-Person Games.

1959

The relationship between Edgeworth's idea of the contract curve and the core was pointed out by Martin Shubik in his paper Edgeworth Market Games. One limitation with this paper is that Shubik worked within the confines of TU games whereas Edgeworth's idea is more appropriately modelled as an NTU game.

1959

Contributions to the Theory of Games IV, A. W. Tucker and R. D. Luce eds., published.

1959

Publication of Martin Shubik's Strategy and Market Structure: Competition, Oligopoly, and the Theory of Games. This was one of the first books to take an explicitly non-cooperative game theoretic approach to modelling oligopoly. It also contains an early statement of the Folk Theorem.

Late 50's

Near the end of this decade came the first studies of repeated games. The main result to appear at this time was the Folk Theorem. This states that the equilibrium outcomes in an infinitely repeated game coincide with the feasible and strongly individually rational outcomes of the one-shot game on which it is based. Authorship of the theorem is obscure.

1960

The development of NTU (non-transferable utility) games made cooperative game theory more widely applicable. Von Neumann and Morgenstern stable sets were investigated in the NTU context in the Aumann and Peleg paper Von Neumann and Morgenstern Solutions to Cooperative Games Without Side Payments.

1960

Publication of Thomas C. Schelling's The Strategy of Conflict. It is in this book that

Schelling introduced the idea of a focal-point effect.

1961

The first explicit application to evolutionary biology was by R. C. Lewontin in Evolution and the Theory of Games.

1961

The Core was extended to NTU games by R. J. Aumann in his paper The Core of a Cooperative Game Without Side Payments.

1962

In their paper College Admissions and the Stability of Marriage, D. Gale and L. Shapley asked whether it is possible to match m women with m men so that there is no pair consisting of a woman and a man who prefer each other to the partners with whom they are currently matched. Game theoretically the question is, does the appropriately defined NTU coalitional game have a non-empty core? Gale and Shapley proved not only non-emptiness but also provided an algorithm for finding a point in it.

1962

One of the first applications of game theory to cost allocation is Martin Shubik's paper Incentives, Decentralized Control, the Assignment of Joint Costs and Internal Pricing. In this paper Shubik argued that the Shapley value could be used to provide a means of devising incentive-compatible cost assignments and internal pricing in a firm with decentralised decision making.

1962

An early use of game theory in insurance is Karl Borch's paper Application of Game Theory to Some Problems in Automobile Insurance. The article indicates how game theory can be applied to determine premiums for different classes of insurance, when required total premium for all classes is given. Borch suggests that the Shapley value will give reasonable premiums for all classes of risk.

1963

O. N. Bondareva established that for a TU game its core is non-empty iff it is balanced. The reference, which is in Russian, translates as Some Applications of Linear Programming Methods to the Theory of Cooperative Games.

1963

In their paper A Limit Theorem on the Core of an Economy G. Debreu and H. Scarf generalised Edgeworth, in the context of a NTU game, by allowing an arbitrary number of commodities and an arbitrary but finite number of types of traders.

1964

Robert J. Aumann further extended Edgeworth by assuming that the agents constitute a (non-atomic) continuum in his paper Markets with a Continuum of Traders.

1964

The idea of the Bargaining Set was introduced and discussed in the paper by R. J. Aumann and M. Maschler, The Bargaining Set for Cooperative Games. The bargaining set includes the core but unlike it, is never empty for TU games.

1964

Carlton E. Lemke and J.T. Howson, Jr., describe an algorithm for finding a Nash equilibrium in a bimatrix game, thereby giving a constructive proof of the existence of an equilibrium point, in their paper Equilibrium Points in Bimatrix Games. The paper

also shows that, except for degenerate situations, the number of equilibria in a bimatrix game is odd.

1965

Publication of Rufus Isaacs's Differential Games: A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization.

1965

R. Selten, Spieltheoretische Behandlung eines Oligopolmodells mit Nachfragetraegheit. In this article Selten introduced the idea of refinements of the Nash equilibrium with the concept of (subgame) perfect equilibria.

1965

The concept of the Kernel is due to M. Davis and M. Maschler, The Kernel of a Cooperative Game. The kernel is always included in the bargaining set but is often much smaller.

1966

Infinitely repeated games with incomplete information were born in a paper by R. J. Aumann and M. Maschler, Game-Theoretic Aspects of Gradual Disarmament.

1966

In his paper A General Theory of Rational Behavior in Game Situations John Harsanyi gave the, now, most commonly used definition to distinguish between cooperative and non-cooperative games. A game is cooperative if commitments--agreements, promises, threats--are fully binding and enforceable. It is non-cooperative if commitments are not enforceable.

1967

Lloyd Shapley, independently of O.N. Bondareva, showed that the core of a TU game is non-empty iff it is balanced in his paper On Balanced Sets and Cores.

1967

In the articleThe Core of a N-Person Game, H. E. Scarf extended the notion of balancedness to NTU games, then showed that every balanced NTU game has a non-empty core.

1967-68

In a series of three papers, Games with Incomplete Information Played by 'Bayesian' Players, Parts I, II and III, John Harsanyi constructed the theory of games of incomplete information. This laid the theoretical groundwork for information economics that has become one of the major themes of economics and game theory.

1968

The long-standing question as to whether stable sets always exist was answered in the negative by William Lucas in his paper A Game with no Solution.

1969

David Schmeidler introduced the Nucleolus in this paper The Nucleolus of a Characteristic Game. The Nucleolus always exists, is unique, is a member of the Kernel and for any non- empty core is always in it.

1969

Shapley defined a value for NTU games in his article Utility Comparison and the Theory of Games.

1969

For a coalitional game to be a market game it is necessary that it and all its subgames have non-empty cores, ie: that the game be totally balanced. In Market Games L. S. Shapley and Martin Shubik prove that this necessary condition is also sufficient.

1972

International Journal of Game Theory was founded by Oskar Morgenstern.

1972

The concept of an Evolutionarily Stable Strategy (ESS), was introduced to evolutionary game theory by John Maynard Smith in an essay Game Theory and The Evolution of Fighting. The ESS concept has since found increasing use within the economics (and biology!) literature.

1973

In the traditional view of strategy randomization, the players use a randomising device to decide on their actions. John Harsanyi was the first to break away from this view with his paper Games with Randomly Disturbed Payoffs: A New Rationale for Mixed Strategy Equilibrium Points. For Harsanyi nobody really randomises. The appearance of randomisation is due to the payoffs not being exactly known to all; each player, who knows his own payoff exactly, has a unique optimal action against his estimate of what the others will do.

1973

The major impetus for the use of the ESS concept was the publication of J. Maynard Smith and G. Price's paper The Logic of Animal Conflict.

1973

The revelation principle can be traced back to Gibbard's paper Manipulation of Voting Schemes: A General Result

1974

Publication of R. J. Aumann and L. S. Shapley's book Values of Non-Atomic Games. It deals with values for large games in which all the players are individually insignificant (non-atomic games).

1974

R. J. Aumann proposed the concept of a correlated equilibrium in his paper Subjectivity and Correlation in Randomized Strategies.

1975

The introduction of trembling hand perfect equilibria occurred in the paper Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games by Reinhard Selten. This paper was the true catalyst for the 'refinement industry' that has developed around the Nash equilibrium.

1975

E. Kalai and M. Smorodinsky, in their article Other Solutions to Nash's Bargaining Problem, replace Nash's independence of irrelevant alternatives axiom with a monotonicity axiom. The resulting solution is known as the Kalai-Smorodinsky solution.

1975

In his paper Cross-Subsidization: Pricing in Public Enterprises, G. Faulhaber shows that the set of subsidy-free prices are those prices for which the resulting revenue ($r_i = p_i q_i$ for given demand levels q_i) vector lies in the core of the cost allocation game.

1976

An event is common knowledge among a set of agents if all know it and all know that they all know it and so on ad infinitum. Although the idea first appeared in the work of the philosopher D. K. Lewis in the late 1960s it was not until its formalisation in Robert Aumann's Agreeing to Disagree that game theorists and economists came to fully appreciate its importance.

1977

S. C. Littlechild and G. F. Thompson are among the first to apply the nucleolus to the problem of cost allocation with their article Aircraft Landing Fees: A Game Theory Approach. They use the nucleolus, along with the core and Shapley value, to calculate fair and efficient landing and take-off fees for Birmingham airport.

1981

Elon Kohlberg introduced the idea of forward induction in a conference paper Some Problems with the Concept of Perfect Equilibria.

1981

R. J. Aumann published a Survey of Repeated Games. This survey firstly proposed the idea of applying the notion of an automaton to describe a player in a repeated game. A second idea from the survey is to study the interactive behaviour of bounded players by studying a game with appropriately restricted set of strategies. These ideas have given birth to a large and growing literature.

1982

David M. Kreps and Robert Wilson extend the idea of a subgame perfect equilibrium to subgames in the extensive form that begin at information sets with imperfect information. They call this extended idea of equilibrium sequential. It is detailed in their paper Sequential Equilibria.

1982

A. Rubinstein considered a non-cooperative approach to bargaining in his paper Perfect Equilibrium in a Bargaining Model. He considered an alternating-offer game were offers are made sequentially until one is accepted. There is no bound on the number of offers that can be made but there is a cost to delay for each player. Rubinstein showed that the subgame perfect equilibrium is unique when each player's cost of time is given by some discount factor delta.

1982

Publication of Evolution and the Theory of Games by John Maynard Smith.

1984

Following the work of Gale and Shapley, A. E. Roth applied the core to the problem of the assignment of interns to hospitals. In his paper The Evolution of the Labour Market for Medical Interns and Residents: A Case Study in Game Theory he found that American hospitals developed in 1950 a method of assignment that is a point in the core.

1984

The idea of a rationalizability was introduced in two papers; B. D. Bernheim, Rationalizable Strategic Behavior and D. G. Pearce, Rationalizable Strategic Behavior and the Problem of Perfection.

1984

Publication of The Evolution of Cooperation by Robert Axelrod.

1985

For a Bayesian game the question arises as to whether or not it is possible to construct a situation for which there is no sets of types large enough to contain all the private information that players are supposed to have. In their paper, Formulation of Bayesian Analysis for Games with Incomplete Information, J.-F. Mertens and S. Zamir show that it is not possible to do so.

1985-86

Following Aumann, the theory of automata is now being used to formulate the idea of bounded rationality in repeated games. Two of the first articles to take this approach were A. Neyman's 1985 paper Bounded Complexity Justifies Cooperation in the Finitely Repeated Prisoner's Dilemma and A. Rubinstein's 1986 article Finite Automata Play the Repeated Prisoner's Dilemma.

1986

In their paper On the Strategic Stability of Equilibria Elon Kohlberg and Jean-Francois Mertens deal with the problem of he refinement of Nash equilibria in the normal form, rather than the extensive form of a game as with the Selten and Kreps and Wilson papers. This paper is also one of the first, published, discussions of the idea of forward induction.

1988

John C. Harsanyi and Reinhard Selten produced the first general theory of selecting between equilibria in their book A General Theory of Equilibrium Selection in Games. They provide criteria for selecting one particular equilibrium point for any non-cooperative or cooperative game.

1988

With their paper The Bayesian Foundations of Solution Concepts of Games Tan and Werlang are among the first to formally discuss the assumptions about a player's knowledge that lie behind the concepts of Nash equilibria and rationalizability.

1988

One interpretation of the Nash equilibrium is to think of it as an accepted (learned) 'standard of behaviour' which governs the interaction of various agents in repetitions of similar situations. The problem then arises of how agents learn the equilibrium. One of the earliest works to attack the learning problem was Drew Fudenberg and David Kreps's A Theory of Learning, Experimentation and Equilibria, (MIT and Stanford Graduate School of Business, unpublished), which uses an learning process similar to Brown's fictitious play, except that player occasionally experiment by choosing strategies at random, in the context of iterated extensive form games. Evolutionary game models are also commonly utilised within the learning literature.

1989

The journal Games and Economic Behavior founded.

1990

The first graduate level microeconomics textbook to fully integrate game theory into the standard microeconomic material was David M. Krep's A Course in Microeconomic Theory.

1990

In the article Equilibrium without Independence Vincent Crawford discusses mixed strategy Nash equilibrium when the players preferences do not satisfy the assumptions necessary to be represented by expected utility functions.

1991

An early published discussion of the idea of a Perfect Bayesian Equilibrium is the paper by D. Fudenberg and J. Tirole, Perfect Bayesian Equilibrium and Sequential Equilibrium.

1992

Publication of the Handbook of Game Theory with Economic Applications, Volume 1 edited by Robert J. Aumann and Sergiu Hart.

1994

Game Theory and the Law by Douglas G. Baird, Robert H. Gertner and Randal C. Picker is one of the first books in law and economics to take an explicitly game theoretic approach to the subject.

1994

Publication of the Handbook of Game Theory with Economic Applications, Volume 2 edited by Robert J. Aumann and Sergiu Hart.

1994

The Sveriges Riksbank (Bank of Sweden) Prize in Economic Sciences in Memory of Alfred Nobel was award to John Nash, John C. Harsanyi and Reinhard Selten "for their pioneering analysis of equilibria in the theory of non-cooperative games".

2002

Publication of the Handbook of Game Theory with Economic Applications, Volume 3 edited by Robert J. Aumann and Sergiu Hart.

2005

The Sveriges Riksbank (Bank of Sweden) Prize in Economic Sciences in Memory of Alfred Nobel was award to Robert J. Aumann and Thomas C. Schelling "for having enhanced our understanding of conflict and cooperation through game-theory analysis".

2012

The Sveriges Riksbank (Bank of Sweden) Prize in Economic Sciences in Memory of Alfred Nobel was award to Alvin E. Roth and Lloyd S. Shapley "for the theory of stable allocations and the practice of market design".

Game theory concerns the behaviour of decision makers whose decisions affect each other. Its analysis is from a rational rather than a psychological or sociological viewpoint. It is indeed a sort of umbrella theory for the rational side of social science, where 'social' is interpreted broadly, to include human as well as non-human players (computers, animals, plants). Its methodologies apply in principle to all interactive situations, especially in economics, political science, evolutionary biology, and computer science. A Chronology of Game Theory is maintained in Internet by Paul Walker. A more extensive exposition of the history of game theory is presented by Mary and Robert Dimand in [59], Robert McCain in [60], and Robert Aumann in [6]. Introduction. Chapter.