A Constructivist Perspective
On Teaching and Learning Mathematics

by Deborah Schifter

Teachers who begin to base their practice on principles of constructivism should not expect to develop a finished repertoire of behaviors that, once achieved, will become routine, Ms. Schifter warns. There is no point of arrival, but rather a path that leads on to further growth and change.

Anne Hendry is a veteran first-grade teacher in rural western Massachusetts. She describes the way she began a unit on measurement shortly before Thanksgiving in 1990.

> Before school, moving desks and chairs and using masking tape, I outlined the shape of a boat on the classroom floor, 16 by 6 feet. This was to represent the *Mayflower.* I also prepared a scroll for a child to read to the class and then to post on the bulletin board with our initial problem involving measurement on it. I selected one child, instructing him that at math time he would be a messenger from the King bringing an "Edict" to the Pilgrims.
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> When math time arrived, Zeb, the child who was appointed to be the messenger, read the "Edict," which said, "This ship cannot sail until you tell me how big it is." The children were puzzled.
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> "Well, what should we do? Who has an idea?" I asked. Thus our discussion of measurement began. Or I thought it would begin. But there was a period of silence – a long period of silence.
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> What do young children know about measurement? Is there anything already present in their life experiences to which they could relate this problem? I watched as they looked from one to another, and I could see that they had no idea where to begin. Surely, I thought, there must be something they could use as a point of reference to expand on. Someone always has an idea. But the silence was long as the children looked again from one to another, to Zeb, and to me.[1]

To most educators, Hendry's intention to connect her mathematics lesson with the upcoming holiday would appear unexceptionable. They would surely wonder, however, at a veteran teacher's choosing to endure so long and painful a silence from a roomful of confused students. Why had she given her first-graders a task without showing them how to complete it? Why ask a question before telling one's students what they need to know in order to answer it? How could so experienced a teacher allow her lesson to falter in this way? Here's Hendry again.

> I was having second thoughts about the enormity of the problem for a firstgrader when, shyly, Cindy raised her hand. "I think it's three feet long," she said. "Why?" I asked. "Because the letter from the King said so," she responded. "I don't understand," I said. "Can you tell me why you think the ship is three feet?" "Because the King's letter said so. See!" Cindy said. "I'll show you."
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> [When the letter was held up], the light, filtering through the paper, made the capital "E" I had written for the word "Edict" look like a three to some of the children. I clarified this point with her and the others who agreed that they also had seen a three on the King's paper. The King, they thought, already knew the answer.
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> I felt we were back to square one again with more silence.[2]

Casting about for a response to their teacher's question, a few of the children look for a number, any number, to connect with the context. But once Cindy's confusion has been cleared up, silence descends again, and Hendry's doubts deepen.

Anne Hendry's behavior will no doubt puzzle readers whose images of teaching derive from the mathematics classrooms in which they themselves once sat as children: the teacher shows her students procedures for getting right answers and then monitors them as they reproduce those procedures. To ask students a question without having previously shown them how to answer it is actually considered "unfair." However, viewed against the backdrop of a "constructivist" perspective on how learning takes place, on what "doing" mathematics can mean, and on what these processes imply for mathematics instruction, Hendry's behavior becomes comprehensible. Indeed, this vignette will allow me to launch an examination, first, of some aspects of instructional practice in mathematics that is informed by such a perspective and, second, of one teacher's experience in constructing such practice. Specifically, I begin by contrasting Hendry's lesson with one taught by Karen Schweitzer, a second-grade teacher. The lesson I describe comes from the interval before Schweitzer began to transform her own mathematics instruction. Then I pick up Schweitzer's story and follow her from the initial phases of her transformation through her inaugural attempts to put that vision into practice. As she articulates her new insights, Schweitzer reveals additional dimensions of the new mathematics pedagogy.

Two Mathematics Lessons

Let us resume with Hendry's account of her measurement unit. Having explained to the class that what they thought was the numeral *3* was actually the letter *E* in "Edict," she continues to describe the children's interactions.

> [Then] Tom raised his hand and said. "Mrs. Hendry, I know it can't be

three feet because the nurse just measured me last week and said that I was four feet, and this boat is much bigger than me!"

From Tom's initial observation, our discussion on measurement was basically off the ground. Hands immediately went up. The children now realized that they knew a little about measurement, especially in relationship to their own size and how tall they were.

"Let's see how many times Tom can fit in the boat," someone suggested. Tom got down and up several times along the length of the boat: the children decided that the boat was four "Toms" long.

"How can we tell that to the King, since he does not know Tom?" I asked. "Send Tom to the King" was their easy solution, while others protested that they wanted Tom to stay on the boat for the trip. I was really hoping that they would relate to the information Tom had already given us about his size. I thought someone might add four feet, four times, presenting us with a quick solution to the problem. But this was not the route they decided to take.

Mark raised his hand and suggested that we could measure the boat with our hands like they do with horses. His neighbor had a horse that was 15 hands [high]. Then we could tell the King how many 'hands' long the boat was. The children agreed that this might be a better idea.

"All right," I said. "Since it was Mark's idea, he can measure the length of the boat with his hands." Mark was also the biggest child in the class.

At first, Mark randomly placed his hands on the tape from one end to the other, but when he double-checked, he came out with a different answer. The children were puzzled for a while as to why this happened. It took several more tries and much discussion before they came to an important conclusion. The children decided that it was necessary for Mark to make sure that he began exactly at the beginning of the boat and did not leave any gaps in between his palms and his fingers as he placed them on the tape. Measuring this way, he discovered the boat was 36 hands long.

Great! We decided to tell the King this, but just to be sure, I suggested we have Sue, the smallest child in the class, measure the other side. She did and related to the class that her side was 44 hands long. Now there was confusion.

"Why are they different?" I asked. "Can we use hands to measure?" "No," the children decided, this would not work either, since everyone's hands were not the same size.

Al suggested using feet. We tried this, but once again, when someone else double-checked with their feet, we found two different measurements. The children at this time began to digress a little to compare each other's hands and feet to discover whose were the biggest and smallest.

Finally, our original discussion continued, while the children explored various concepts and ideas. Joan sat holding a ruler, but, for some reason, did not suggest using it. Perhaps, I thought, it might be that her experience with a ruler was limited, and she may not have been quite sure how to use it.

Our dilemma continued into the next day when the children assembled again to discuss the problem with some new insights. One child suggested that since Zeb knew the King, and everyone knew Zeb, we should use his foot. "Measure it out on a piece of paper and measure everything in 'Zeb's foot.'" Using this form of measurement, the children related to the King that the boat was 24 "Zeb's foot" long and nine "Zeb's foot" wide.

Curiosity began to get the best of them, and the children continued to ex plore this form of measurement by de ciding to measure each other, our class room, their desks, and the rug using "Zeb's foot. " I let them investigate this idea for the remainder of the math period.

On the third day of our exploration, I asked the children why they thought it was important to develop a standard form of measurement (or in words understandable to a first-grader, a measurernent that at would always be the same size) such as using only '"Zeb's foot" to measure everything. Through the discussions over the past several days, the children were able to internalize and verbalize the need or importance for everyone to measure using the same instrument. They saw the confusion of using different hands, bodies, or feet because of the inconsistency of size.[3]

Hendry goes on to describe how her class arrived at an exploration of the use of rulers and the adoption of conventional units of measurement. Yet some important aspects of her teaching are already available to us.

First, we do not see Hendry engaged in the commonest of traditional teaching behaviors – giving directions and offering explanations. Instead, we observe her questioning her students, and the questions sometimes come minutes apart. When they do come, more often than not they appear to elicit, rather than allay or forestall, confusion.

To look more deeply at Hendry's lesson, let me now contrast it to one described by Karen Schweitzer, who also works with small groups of children in a rural setting in western Massachusetts. While her class was doing a science unit on whales, the children became so fascinated by the fact that blue whales can grow to 100 feet in length that Schweitzer decided to have them measure out that length in the hallway.

> I told the children exactly how we would go about measuring the whale's length. We would take the yardstick, which we hadn't explored, and we would put it down and keep track of where it ended and then place it there and keep counting till we reached where it ended and then place it there and keep counting till we reached 100 feet.[4]

After they were done, Schweitzer reported, the children ran up and down the hall exclaiming, "Wow!" However, despite the evident pleasure they derived from the results, she herself felt somewhat unsatisfied with the lesson. What, if anything, she wondered, had the children learned? The similarities between the two lessons are easily identified. Both Hendry and Schweitzer were responsive to what had captured their students' imaginations – Hendry's class had been fascinated by a cutaway of the *Mayflower* she had made;

Schweitzer's, by the length of the blue whale. Both teachers decided to engage the class in measurement activities connected to the topics, and both teachers set up their lessons to involve the children in the actual measuring – their lessons were "hands-on."

From the point of view of this discussion, however, the salient difference is that, while Schweitzer told her class exactly how to perform the task she had devised, Hendry posed a problem and expected the children to find their own way to a solution. Schweitzer crisply demonstrated the use of a yardstick; Hendry watched her students messily struggle to figure out what the inconsistencies in their results could tell them about the concept of measurement. While Schweitzer could have demonstrated the procedure to any number of students, Hendry's lesson depended on her students interacting among themselves.

From these two units on measurement, what can we infer about the epistemological assumptions that underlie them? "Hands-on" though it may have been, Schweitzer's lesson was nonetheless consistent with beliefs about learning that still order most of our classrooms: that people acquire concepts by receiving information from other people who know more; that, if students listen to what their teachers say, they will learn what their teachers know; and that the presence of other students is incidental to learning.

However, although Schweitzer's students might now have been able to picture just how long a blue whale can get, most, she realized, had probably learned very little about the concept of measurement. They had not had an opportunity to think through as a group what a yardstick is, or why it was important to place it exactly as Schweitzer had shown them.

An alternative perspective on learning mathematics in the K-12 classroom is implied in Hendry's lesson. It is a perspective that informs the principles guiding the current movement for mathematics education reform: that individuals necessarily approach novel situations by interpreting them in the light of their own established structures of understanding; that the construction of new concepts is provoked when those settled understandings do not satisfactorily accommodate a novel circumstance; and that this constructive activity is not simply an individual achievement but one embedded in and enabled by contexts of social interaction.[5]

Paralleling this divergence in epistemological assumptions is a fundamental difference in how the nature of mathematics and the "doing" of mathematics are understood. The drill-and-practice approach to math instruction has an affinity for a static and timeless conception of mathematical truth ("all the mathematics there is has always already been out there"). The constructivists, on the other hand, argue that mathematics is a human invention with a long history; culturally embedded schools of thought compete, fashions change, and some questions may be irresolvable. Until quite recently, it was the apparent certainty of mathematics that raised its status above that of the natural sciences. Today, a keener appreciation of the interplay of imagination and logical necessity, coupled with a greater awareness of the roles of convention, philosophical commitment, and technological interest in shaping the development of the discipline, favors an emphasis on the similarities between mathematics and the natural sciences. In this view, then, to "do" mathematics is to conjecture – to invent and extend ideas about mathematical objects – and to test, debate, and revise or replace those ideas.[6]

In the 1980s these new perspectives on the learning process and on the nature of mathematics converged to create a drastically revised vision of what should be taking place in the classroom. Teaching mathematics was reconceived as the provision of activities designed to encourage and facilitate the constructive process. The mathematics classroom was to become a community of inquiry, a problem-posing and problem-solving environment in which developing an approach to thinking about mathematical issues would be valued more highly than memorizing algorithms and using them to get right answers. Students would learn how to construct a mathematical argument and assess its mathematical validity.[7]

Returning to Anne Hendry's first-graders, we see that their teacher has posed a problem – report to the King the size of the *Mayflower* – in order to launch them on an exploration of the basic concepts of measurement. As the children make suggestions about the length of the boat, Hendry does not indicate whether they are right or wrong. Instead, she listens and watches. Only when the children seem satisfied with a solution does she pose a further question, which leads them to yet another problem, their own problem, which they feel compelled to solve.

As Hendry sees it, her task is to pose questions that will lead through – rather than around – puzzlement to the construction of important mathematical concepts. Thus, when the class decides the boat is four "Toms" long, she points out that since the King does not know Tom, he cannot know how long the boat is. Once they have agreed that the boat is 36 hands long, she suggests that a second child measure it, knowing that this will ignite again their puzzlement and drive them toward a more durable solution and a deeper grasp of the concept of measurement. The children un derstand that in their mathematics lessons it is up to them to offer their thoughts about the questions that are posed and to find a resolution for any contradictions that arise.

During the first days of the measurement project, the children are involved in making meaning out of the activity of measuring the boat. By the third day, Hendry concludes, the children have figured out that in order to say how long something is, they must count the number of times an agreed-upon unit is laid down, without overlap or gaps, to cover the length of the object. And they know that they must be able to communicate the size of the unit as well as the number of units counted.

Constructing a New Vision

Karen Schweitzer writes that, having taught kindergarten for a number of years, she became anxious when she first found out that she would be teaching second grade. "I was eager to stretch my kindergarten language-arts program to meet the needs of the second-graders that I would be facing, but how would I teach math?" she wondered. Previous inservice work had stimulated her thinking about how children learn.

> [Tbey] need to be invested in what they do,... they need to work at their own levels of understanding, and ... it is important for them to have practice in not only skill areas, but in identifying the strategies they are using as readers and writers.[8]

However, like so many other thoughtftul teachers, Schweitzer's ideas about mathematics and mathematics instruction did not allow her to translate these beliefs about how children learn to use language into her teaching of mathematics. Since she believed that children learn best by doing, her classroom was well-stocked with a variety of mathematics manipulatives. But she never encouraged the children to explore for themselves; they used these materials in just the ways she prescribed, in the ways that made sense to her.

As she fretted over her new mathematics program, she concluded that what she needed was for someone to tell her what activities to do. So she gathered suggestions for activity books that would help her. "The books had interesting ideas and some fun games," she recalls, "and between those books, and a few conversations with my colleague, I made it to spring." Yet, because she was dissatisfied with the results of her searching, she decided to attend a two-week summer institute for teachers of mathematics.

> I thought, "That's it. This year I really have to do this. But it'll be okay. I'll go to [the institute], they'll tell me what to do, and my worries will be over." I thought that I would . . . get a list of problems for my second-graders to solve, or at least a recipe for how to write them, and a list of questions to ask the kids. Then my math program would be all set.[9]

Like most teachers who attend inservice programs, Schweitzer expected to be told how to teach. After all, this is what such experiences have traditionally been designed to do. But when the goal is to develop a practice based on ideas about the nature of mathematics and about how people learn, such as I have been describing, telling teachers what to do simply isn't useful.

To see why this is so, let us consider Schweitzer's desire for lists of problems and questions and whether Hendry's lesson offers any clues as to how such lists might be created. Hendry did start with a nice problem. In itself, though, this exercise is no more promising than Schweitzer's project. And what were the questions that drove Hendry's lesson? "What should we do?" "Who has an idea?" "Why?" "I don't understand Can you tell me why the ship is three feet?" "How can we tell that to the King, since he does not know Tom?" Clearly, the central feature of Hendry's lesson was neither the problem she posed nor the specific questions she put, but rather the nature of the discussion her students engaged in – and which she skillfully guided. Hendry's practice cannot be scripted. It depends on the teacher's capacity to respond spontaneously to students' perplexities and discoveries.

A teacher development program that simply provides participants with a repertoire of teaching strategies and techniques would be like Schweitzer's lesson on measurement, in which her students were shown how to place a yardstick without ever understanding why they should do it in a particular way. In the end, the classroom structures Hendry chose and the questions she asked were all subordinated to the mathematics she wanted her students to learn and to her understanding of how they would best learn it. The subtleties of timing and tone and the appropriateness of her spontaneous responses stem from a coherent conception of what should be happening in her classroom.

One important way to help teachers develop new conceptions of what can happen in their classrooms is to allow them to experience as students classrooms in which the new approach to teaching is enacted, classrooms that provide learning experiences powerful enough to challenge 16 and more years of traditional education. Teachers must be able to recognize for themselves that this is the kind of learning that they would choose to foster in their own classrooms, and they must be given opportunities to analyze the process of learning, the nature of mathematics, and the kinds of classroom structures that will promote that goal.

Through mathematics lessons that challenge teachers at their own levels of mathematical competence, teachers can increase their mathematical knowledge and can experience a depth of learning that is, for many of them, unprecedented. Such activities allow teachers to encounter mathematics, often for the first time, as an activity of construction, exploration, and debate, rather than as a finished body of knowledge to be accepted, accumulated, and reproduced.

The institute Schweitzer attended provided just such experiences. Along with 35 other elementary teachers, she took on the challenge to devise a number system using the symbols O, A, B, C, and D, in which one can add, subtract, multiply, and divide. In another exercise, she and her colleagues examined the angles of polygons, looked for patterns, made conjectures, argued about why the patterns they found must hold for all polygons, and, in the environment of Logo's turtle geometry, learned to pose questions of their own and then to work to find their own answers.

After several days of such active, verbalized problem solving, Schweitzer and her colleagues concentrated on listening. Working with videotapes and in live, one-on-one interviews, they analyzed students' solution processes, assessing the extent of the students' understandings and exploring the significance of the gaps in understanding that were exposed. Throughout the institute, the participants were asked to reflect on their own experiences. There were frequent small- and whole-group discussions about what had just happened in a lesson or what was seen on videotape; participants kept joumals; and they reflected in writing on such fundamental questions as "How have your ideas about learning changed?" or "What does it mean to understand mathematics?"[10]

As the summer institute ended, Schweitzer wrote about what she was learning.

> I have been greatly influenced by my observations of myself as a learner and the implications that my learning has for my teaching. For example, I have learned that I need to work hard not to shut down when I get the answer to something, but to do the hard work of asking myself more questions after I think I've found the answer. But how does/did the shutting down behavior influence my teaching? Is that what makes it so hard for me to see what the probing questions are? Do I also shut down when a student gets the right answer? I am rediscovering the discipline to make myself wonder. I know that sounds contrary – discipline and wonder – but until it becomes habit again, it will require discipline.
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> I have looked carefully at the processes that I use to, teach. I have usually taught with the methods and materials that make sense to me. I have used manipulatives and other useful aids to show [the children] my way of understanding. My work now is to let them find their way of understanding. A habit of thinking that I have been thinking about is changing who controls the child's exploration processes. It was such a powerful experience for me to be able to pose my own question/investigation in Logo and to keep myself puzzling over it....
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> I came to the institute with the goal of learning how to ask questions. What I learned was why I need to ask questions – they help a learner to find words for her/his thinking, they help "get the thinking out". . . What a big and subtle difference from the past when I think I asked questions after the fact to recap what [students] had done. The questions sometimes told me what [the children] did, but they didn't help them find a new exploration, and they didn't ask for a change of control....
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> I want to remember to affirm the ideas that I came with that have been reinforced here. I feel even more certain than I did before that it needs to be explored that math is part of everything around us and that it is like a language. I continue to want my students to associate math with playing and puzzles, but now I think it's more. It's the working at wondering and the delight in wondering![11]

The institute did not give Schweitzer the recipe for writing word problems or the list of questions she had initially hoped to receive. Instead, she was given the opportunity to construct a new vision for her mathematics teaching – a vision grounded in an enlarged sense of mathematics, as well as in her own analysis of the kind of learning she wanted for her classroom.

Constructing a New Practice

Creating a teaching practice guided by constructivist principles requires a qualitative transformation of virtually every aspect of mathematics teaching. The development of a new vision is only the first step. Like many of the participants, Schweitzer left the summer institute with a high level of excitement and anticipation, along with some trepidation. The greatest challenges still lay ahead.

Join her now, in early September, just weeks after her participation in the institute.

> The first time we had math was during the second day of school. I had intended to start my math program on the very first day with such an exciting and inviting math activity that the children would be captivated and hooked on math for the rest of the year. I wanted to be inspired and to create this magnificent problem by myself – to apply all the things that I had learned in the institute. But I never quite figured out what that perfect activity was, so I just skipped math that day...
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> On the second afternoon, I announced that it was time for math. One little girl said, almost as if she was asking per mission to be excused, "I don't really want to do math." I knew that she was talking about "papers" with counters and endless problems, and so I answered, "I think second-grade math will be quite a bit different from first-grade math, so don't worry yet." She looked momentarily appeased, but her face told me that she wasn't quite ready to believe me. We all sat down on the carpet in the meeting area, and several more children voiced their dislike for math. I again said, "Wait. Don't decide yet. See what it's like first." After hearing what they thought of "math," l really felt the pressure to do something that met all my criteria for the perfect math lesson. So I took a deep breath and said, "I noticed that some of the things that you wondered about yesterday sounded like they had to do with math." "Yeah," agreed John, and he proceeded to read all of the "how many" questions from the chart [where we had listed questions the day before]. "So much for discussion," I thought, "and where is the probing question in here anyway?" But I figured that I had to go somewhere with it and added, "Why are those math questions?"
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> John answered, "Because they are counting. "
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> "Is math only counting?" The children slowly began to throw out ideas about what math was.
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> "It's like putting 3 and 4 together."
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> "It's when you put unifix [cubes] together to add."
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> "Could math be patterns?" I asked. "Uh-huh," they agreed, and they began to make patterns with the unifix cubes that were in the middle of the circle. I made sure to emphasize and affirm all of the different ideas.
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> I then returned to the question of how many books were in the room. (I chose that item mostly because I thought it couldn't hurt to focus our attention on books.) "How could we answer that question?" I asked. "It's easy, just count them," they told me. I said that we were going to get into groups of three and figure out lots of different ways that we could count the books.... Then we would get back together and share all the different ideas, and after that we would count the books.
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> I really thought that all was going well now. They would go

> through this process, I would ask lots of great questions, they'd learn a lot, and we'd be off to a great start. Exoept, when I mentioned breaking up into groups, all of a sudden, two children had stomachaches, and one child started to sneeze. "But we can't stop here, before we really get started" I thought, and I pushed on.
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> When the children finally settled down in their groups, they were all quiet and seemed shy about talking to each other in this way. I hadn't expected this task to be as difficult as it was. And when we got back together and they shared their ideas (count by 2s, by 5s, by 10s), I found that the probing question of "How would you do that?" fell flat on its face. We had been working for 35 minutes, and I decided to leave the actual counting until the next day, so I recapped how many different ideas there were, and that was our first day of math.
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> I ended that day frustrated and disappointed. I had wanted to dazzle them, to show them that math is interesting and inviting. Instead, we all ended up a little unsure by the end of the lesson. Our nerves were showing, theirs and mine, and none of us were sure what lay ahead this year.[12]

In fact, the session that Schweitzer described might not have been such a bad start to the year. Both she and her students had to learn a new way of being in math class, and at first they felt rather unsteady. The children didn't yet know how to talk to one another in their small groups, Schweitzer didn't yet know how to interpret her students' comments, and neither she nor the students had figured out how to have mathematical discussions.

But a process had begun. Schweitzer had communicated the expectation that mathematics was something to talk about and that the children were to discuss their own ideas with her and with one another. At this early stage, though, Schweitzer lacked the perspective that would have aIlowed her to trust that the process would develop. Her writing from two weeks later shows her frustration.

> I am so frustrated that lately I've just wanted math to go away! However, I put math in a prominent and unavoidable place in my daily schedule this year so that I couldn't slip past it. So now, there it is. Every day. Waiting to taunt Me.
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> I've tried to create activities that were engaging and meaningful, but the children seem inattentive during discussions, and my questons are often answered with silence. I've tried to use resource books to set up activities that are "proven" in order to stimulate thinking and talking, but nothing happens except that I get even more frustrated. So I try to listen to the kids for a direction to go in, but I guess I don't know what I'm hearing yet because that doesn't help me either.[13]

How does one move from the state that Schweitzer has described – frustrated and unsure, unable to identify or assess progress, feeling that things aren't right but not knowing how to make them better – to a coherent practice? Persistence and patience are part of the answer; there is no mystery there. But another key component is the opportunity to reflect on each day's events.

It is widely recognized that, as teachers return to their classrooms from summer institutes or intensive workshops, animated by a fresh sense of possibilities, the provision of continued support is crucial to realizing those possibilities in day-to-day instruction.[14] Various programs offer different kinds of structures: clinical supervision, biweekly seminars, study groups, full-day retreats. Critical to each is the opportunity to think through events from one's own classroom in the light of new goals, beliefs, and understandings.

In Schweitzer's case, support came in the form of a writing course. Nineteen teachers (with varying levels of experience in teaching based on constructivism) were invited to meet weekly with an inservice educator to write about what was happening in their own mathematics classrooms.

The course focused on two major activities: reading assigned materials and writing. The reading materials included reflective pieces written by teachers about their own mathematics instruction and articles coming out of the current movement to reform the teaching of reading and writing.[15] In addition to such works, Schweitzer's class also readpapers written by two groups of teachers who had earlier participated in the same course. All the readings were critically examined for both content and writing style.

The writing component of the course was fashioned after the process-writing model that many of the elementary teachers already used in their own classrooms. Consistent with the new mathematics pedagogy, process writers work cooperatively to analyze and edit their projects. For the first several weeks of the course, specific assignments were given so that teachers could explore pedagogical issues and experiment with writing styles. Eventually, teachers determined the direction of their own writing and worked on final projects – 15 to 40-page reflective narratives on topics of their choosing. Throughout the course, teachers met in both small and large groups to share their works in progress and to solicit feedback. All work was turned in to the instructor, who responded in writing.

In asking Schweitzer and her colleagues to reflect on their classroom processes, the course offered them the opportunity to track the development of their new practice. Schweitzer discovered how, through writing, she could revisit a lesson, "listen" again to what her students were saying, and consider how what she now heard in their words might influence her instruction. She found the requirement that descriptions of classroom events be made understandable to others especially helpful, for it required her to make sense of those events for herself. For example, after an extremely frustrating and confusing lesson, she reported that she had spent hours at her journal, trying to sort out what had happened.

However, once she took it upon herself to write a narrative about the lesson -- making the sequence of events that had thwarted her comprehensible to an audience -- she felt better about it. "Although I ended saying I was frustrated," she told me, "I wasn't feeling it as passionately as I was when I started. The writing of it cleared things up for me. I saw learning and a continuity that I didn't (and couldn't) see even after writing in my journal."

Weekly meetings with other teachers and feedback from her instructor allowed Schweitzer further opportunities to reflect on her developing practice. She shared with her colleagues what had been happening in her classroom and entertained their interpretations of those events. As a group, these teachers considered what her students were learning, where they might be confused, and what she might try next. Through similar discussions of her colleagues' stories, she could picture other groups of children, working in different classroom structures, and could learn about the results of pedagogical decisions she herself might not have made. In addition, she learned from her group the very important lesson that her frustrations and feelings of incompetence were not unique.

I hoped that I wouldn't feel embarrassed when others read about what was happening in my classroom and how hard a time I was having. When I brought this to class, I was relieved and encouraged to find out that many other teachers had similar feelings of frustration.[16]

In fact, after several more weeks, Schweitzer began to see progress in the way she worked with her students. Not only that, but she had also become more tolerant of herself when her lesson did not meet her expectations. After one lesson that didn't go well, she was surprised to find that she didn't feel that combination of frenzy, frustration, and hopelessness that she had felt five or six weeks earlier.

By using my journal as a tool, I was able to figure out that, if that many children were not on task, there was something wrong with either the structure or the content of the activity and I needed to reexamine both aspects of what we had done before I decided what to do next. . . .

Tonight, I knew that the answers lay in my reflection, not in a book or in a manual. I know now that two of my students, because of their special language needs, and two other students (in addition) have difficulty working in pairs. Maybe it's time to try groups of three to take some of the pressure off of the individual kids. But then how do I keep them involved and accountable? Create an activity that has a task for each child and see how that works out. It sounds good in theory, but the only way I'll know is to try it and to watch how it works.[17]

The previous summer Schweitzer had adopted as a goal for her teaching that her students learn to try out ideas, testing them to see if they worked and modifying them in the light of their analyses. Now she was learning that the same process applied to her own practice. Not every lesson would be dazzling. The children would not always be captivated and engaged in fascinating discussions. As she worked to deepen her new practice, she too would need to try out ideas, test them to see if they worked, analyze the results, and develop her plans accordingly. And now she had confidence that she could figure out what was going on and come up with new things to try.

Three months into the school year, as Schweitzer looked back on what she had gone through, she noted her progress.

> One of the most striking things that I have learned about math education is that, at least for the time being, there isn't a source that has the answers neatly prepackaged for me. A year ago I was in search of the perfect resource book to tell me what to do. I looked for answers from someplace else. Now instead, I think, "What are my students thinking about? What interests second-graders? What concepts are they ready to explore?" and even "What do these children need to learn in second grade?" Instead of looking for the authorities who "know it all" to tell me the answers, I now look to well-trained, experienced teachers to help me interpret what is happening and to use their experience to offer suggestions.
>
> I have had a few revelations about the direction of my curriculum during this time. It was a big leap to realize that I didn't need to present the curriculum in the order it's always done in second grade. I remember sitting in my writing class one night and saying to my small group, "You mean I don't have to teach adding, then subtracting, then trading?" At some point, after I decided that the direction of the curriculum had to come from the children, I also realized that it was okay for me to say, "Ah, this is what they are ready to do now. I can present a challenge to them, or introduce a topic, and look for their questions within that area, for there will surely be some."
>
> The ... Writing Project has played an important role in my course of change. The class gave me a reason to write about what was happening in my classroom, and the writing has had to be clear and understandable because other teachers were going to read it. Part of the way through the class, I realized that writing about my frustrations caused me to reflect on what had happened and even to reflect on earlier reflections. This was part of what was making me feel different about these frustrations.
>
> Meeting with a group of teachers each week helped me not only by giving me feedback on my writing but feedback on the math that was happening in my classroom as well. It also gave me a chance to read and hear about what was going on in other classrooms. Developing this habit of reflecting and sharing has been a pivotal part of my change. It has struck me several times that these are pieces that are often missing from teacher education programs and from our daily professional lives, and, for me, these were pieces that were essential.
>
> I have also realized that I have a lot of work still ahead of me. If there had been easy answers, I would have gotten them and been done with it, but now every question raises a new question, and often even the answers raise questions. I just hope I've teamed enough to figure out how to find the answers and what to do with them once I've found them.[18]

Several months into the school year Schweitzer recognized that she still had a lot of work ahead of her. In fact, she had started down a road that has no endpoint. Teachers who begin this process expecting to develop a finished repertoire of behaviors that, once achieved, will become routine will be disappointed. Teaching this way is necessarily disruptive of routine, for no other reason than that students will continually surprise us with their own discoveries.

For many teachers, this approach implies a change in their relationship to their own profession. Instead of concentrating on technique and strategy – keeping up with the latest trends – the new pedagogy means developing an attitude of inquiry toward classroom processes. In other words, the method that Schweitzer has learned – to try out ideas in the classroom and analyze students' learnings – is not merely the means to her new practice but also its essence. There is no point of arrival, but rather a path that leads on to further growth and change. For those who are willing to face the doubts, frustrations, and uncertainties inherent in a practice based on constructivism, that path is also filled with rewards and satisfactions.

FOOTNOTES

1. Anne Hendry, "Math in the Social Studies Curriculum," in Deborah Schifter, ed., What's Happening in Math Class? Volume 1: Envisioning New Practices Through Teacher Narratives (New York: Teachers College Press, 1996), p. 10.
2. Ibid., p. 11.
3. Ibid., pp. 11-12.
4. Personal communication with Karen Schweitzer, 1993.
5. Paul Cobb, "The Tension Between Theories of Learning and Instruction in Mathematics Education," Educational Psychologist, vol. 23, 1988, pp. 97-103; Jere Confrey, "Learning to Listen: A Student's Understanding of the Powers of Ten," in Ernst von Glasersfeld, ed., Radical Constructivism in Mathematics Education (Dordrecht, The Netherlands: Kluwer Academic Publishers, 1991), pp. 111-38; Constance Kamii, Young Children Reinvent Arithmetic: Implications of Piaget's Theory (New York: Teachers College Press, 1985); and Ernst von Glasersfeld, "An Exposition of Constructivism: Why Some Like It Radical," in Robert B. Davis, Carolyn A. Maher, and Nel Noddings, eds., Constructivist Views on the Teaching and Learning of Mathematics: Journal for Research in Mathematics Education Monograph No. 4 (Reston, Va.: National Council of Teachers of Mathematics, 1990), pp. 19-30.
6. Phillip J. Davis and Reuben Hersh, The Mathematical Experience (Boston: Birkheuser, 1980); Paul Ernest, The Philosophy of Mathematics Education: Studies in Mathematics Education (London: Falmer Press, 1991); Imre Lakatos, Proofs and Refutations (Cambridge: Cambridge University Press, 1976); Magdalene Lampert, "The Teacher's Role in Reinventing the Meaning of Mathematics Knowing in the Classroom," in Merlyn J. Behr, Caroline B. Lacampagne, and Margariete M. Wheeler, eds., Proceedings of the 10th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (DeKalb: Northern Illinois University, 1988), pp. 433-80; Bruno Latour, Science in Action (Cambridge, Mass.: Harvard University Press, 1987); and Thomas Tymoczko, New Directions in the Philosophy of Mathematics (Boston: Birkheuser, 1986).
7. Cobb, op. cit.; Confrey, op. cit.; Davis, Maher, and Noddings, op. cit.; Deborah Loewenberg Ball, "With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics," Elementary School Journal, vol. 93, 1993, pp. 373-97; idem, "Halves, Pieces, and Twoths: Constructing Representational Contexts in Teaching Fractions," in Thomas P. Carpenter, Elizabeth Fennema, and Thomas Romberg, eds., Rational Numbers: An Integration of Research (Hillsdale, N. J.: Erlbaum, 1993), pp. 157-96; Eleanor Duckworth, 'The Having of Wonderful Ideas' and Other Essays on Teaching and Learning (New York: Teachers College Press, 1987); Catherine T. Fosnot, Enquiring Teachers, Enquiring Learners: A Constructivist Approach for Teaching (New York: Teachers College Press, 1989); and Deborah Schifter and Catherine T. Fosnot, Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenge of Reform (New York: Teachers College Press, 1993).
8. Karen Schweitzer, "The Search for the Perfect Resource," in Deborah Schifter, ed., What's Happening in Math Class? Volume 2: Reconstructing Professional Identities (New York: Teachers College Press, 1996), p. 48.
9. Ibid.
10. For further descriptions of such courses, see Deborah Schifter, "Mathematics Process as Mathematics Content: A Course for Teaching Mathematics," Journal of Mathematical Behavior, vol. 12, 1993, pp. 271-83; Schifter and Fosnot, op. cit.; Deborah Schifter and Martin A. Simon, "Assessing Teachers' Development of a Constructivist View of Mathematics Learning," Teaching and Teacher Education, vol. 8, 1992, pp. 187-97; and Martin A. Simon and Deborah E. Schifter, "Towards a Constructivist Perspective: An Intervention Study of Mathematics Teacher Development," Educational Studies in Mathematics, vol. 22, 1991, pp. 309-31.
11. Personal communication with Karen Schweitzer, 1992.
12. Schweitzer, pp. 50-51.
13. Ibid., p. 55.
14. David K. Cohen et al., Effects of State-Level Reform of Elementary School Mathematics Curriculum on Classroom Practice (East Lansing: National Center for Research on Teacher Education and Center for the Learning and Teaching of Elementary Subjects, Michigan State University, Research Report 90-14, 1990); and Bruce Joyce, ed., Changing School Culture Through Staff Development (Alexandria, Va.: Association for Supervision and Curriculum Development, 1990).
15. See Ball, "With an Eye"; idem, "Halves, Pieces"; Ruth M. Heaton, "Continuity and Connectedness in Teaching and Research: A Self-Study of Learning to Teach Mathematics for Understanding," paper presented at the University of Pennsylvania Ethnography in Education Research Forum, Philadelphia, 1991; Magdalene Lampert, "Arithmetic as Problem Solving," Arithmetic Teacher, March 1989, pp. 34-36; and idem, "The Teacher's Role." See also Nancie Atwell, "Writing and Reading from the Inside Out," in Jane Hansen, Thomas Newkirk, and Donald Graves, eds., Breaking Ground: Teachers Relate Reading and Writing in the Elementary School (Portsmouth, N. H.: Heinemann Educational Books, 1985), pp. 147-68; and George Hillocks, Jr., "Teaching, Reflecting, Researching," in Donald A. Daiker and Max Morenberg, eds., The Writing Teacher as Researcher (Portsmouth, N. H.: Heinemann, 1990), pp. 15-29.
16. Schweitzer, p. 55.
17. Ibid., p. 64.
18. Ibid., pp. 64-65.

Constructivist Approach. Constructivism is an epistemology, or a theory, used to explain how people know what they know. The basic idea is that problem solving is at the heart of learning, thinking, and development.Â A constructivist approach to learning and instruction has been proposed as an alternative to the objectivist model, which is implicit in all behaviorist and some cognitive approaches to education. Objectivism sees knowledge as a passive reflection of the external, objective reality.