HISTORIA MATHEMATICA **24** (1997), 445–453
ARTICLE NO. HM972161

REVIEWS

Edited by Catherine Goldstein and Paul R. Wolfson

All books, monographs, journal articles, and other publications (including films and other multisensory materials) relating to the history of mathematics are abstracted in the Abstracts Department. The Reviews Department prints extended reviews of selected publications.

Materials for review, except books, should be sent to the editor of the Abstracts Department, Prof. David E. Zitarelli, Department of Mathematics, Temple University, Philadelphia, PA 19122, U.S.A. Books in English for review should be sent to Prof. Paul R. Wolfson, Department of Mathematics and Computer Science, West Chester University, West Chester, PA 19383, U.S.A. Books in other languages for review should be sent to Prof. Catherine Goldstein, Bat 425 Mathématiques, Université de Paris-Sud, F-91405 Orsay Cedex, France.

Most reviews are solicited. However, colleagues wishing to review a book are invited to make their wishes known to the appropriate Book Review Editor. (Requests to review books written in the English language should be sent to Prof. Paul R. Wolfson at the above address; requests to review books written in other languages should be sent to Prof. Catherine Goldstein at the above address.) We also welcome retrospective reviews of older books. Colleagues interested in writing such reviews should consult first with the appropriate Book Review Editor (as indicated above, according to the language in which the book is written) to avoid duplication.

Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences. Edited by Ivor Grattan-Guinness. London (Routledge). 1994. xiii + 1806 pp.

Reviewed by Victor J. Katz

Department of Mathematics, University of the District of Columbia, Washington, D.C. 20008

A review of an encyclopedia has a purpose different from a review of a textbook or even of a general scholarly book in history. The reader of a textbook review will want to know how the text compares to similar ones so a decision on adoption can be made. And the reader of the review of a book on the history of a particular topic may well want to know about the accuracy and completeness of the coverage to decide whether to invest the $50 or $75 necessary to purchase it or, perhaps, whether to recommend that the university library invest the funds. But an encyclopedia is different. It will be a relatively rare reader who is willing to invest $200 in a reference work of which inevitably only part will be of interest. And besides, the university library can usually be counted on to buy an encyclopedia, precisely because enough people will be interested in at least part of the work for it to have a reasonable rate of usage. Thus a review must address the questions of whether the *Encyclopedia* succeeds as a new reference book, a book that readers will turn to first for basic information about topics in the history of mathematics.

0315-0860/97 $25.00

As Grattan-Guinness indicates, the *Encyclopedia* has two basic purposes. In his introduction, the editor writes that the *Encyclopedia* "tries to recover our mathematical heritage by presenting an introduction to all the main branches, techniques, cultural traditions and applications of mathematics from ancient to modern times." He also expresses the hope that the *Encyclopedia* will be of value to teachers of mathematics at various levels as they attempt to use the history of mathematics in the classroom. So given the stated purposes of the *Encyclopedia*, I will compare it to other reference books on the market which have similar goals. What are these other works? First, there is the four-volume mathematics section of the *Dictionary of Scientific Biography* (New York: Scribner's, 1970–1980). The major problem with that work is that it must be entered with a name. One can learn about all the mathematics accomplished by a particular individual, but it takes a lot of back and forth work using the index to learn something about the history of a particular topic. Second, there is Morris Kline's one-volume *Mathematical Thought from Ancient to Modern Times* (New York: Oxford University Press, 1972). That book has the major disadvantage that it was written by one author, who could not possibly be expert in all fields, and it is inevitably dated, having been written a quarter of a century ago. The Grattan-Guinness *Encyclopedia*, by contrast, was written by 134 different authors, most of whom are currently doing research in their fields of expertise. And third, there are the several textbooks in the history of mathematics, books which though written as texts can also be used for reference even though they are not nearly so comprehensive as either of the other works. So the questions I will attempt to answer are, first, can you get a better introduction to the history of the "main branches" of mathematics in the Grattan-Guinness *Encyclopedia* than in the works already available on your library shelves; and, second, can the teacher trying to use history in the classroom find appropriate material to use with students?

The short answers to the questions just posed are "Yes" for certain areas, but unfortunately "No" for too many important ones. The rest of this review will amplify these short answers by considering topics in the order in which they are presented in the *Encyclopedia.*

What can you learn about Babylonian and Egyptian mathematics? Jens Høyrup presents a brief survey of Babylonian mathematics using the most recent research. Thus, you learn that the algebra of the Babylonians is built on "naive" geometry, based on manipulation of lengths and areas, lacking symbols, and with no formal proof. General methods for problem solving appear only rarely on the tablets; rather the methods which the scribes had worked out are demonstrated through numerous problems designed as paradigms. In addition, though these problems are often stated in "real-life" terms, they are usually artificial problems designed to show the scribes' virtuosity. Unfortunately, the article has no discussion of a major point of debate today, the impact, if any, that Babylonian mathematics may have had on classical Greek mathematics. Nor are there enough examples to allow the prospective teacher to use the material in the classroom.

Clara Roero, on the other hand, does at least present some thought-provoking quotations relating to the influence of ancient Egyptian mathematics on Greece. She also presents enough examples in her survey of the mathematics of the Rhind and Moscow papyri to allow the reader to understand how the Egyptians solved mathematical problems.

Can you get a good introduction to the mathematics of ancient Greece in the *Encyclopedia*? Alexander Jones makes a valiant effort to describe Greek mathematics in two-and-a-half chapters encompassing some 21 pages. We get a whirlwind tour encompassing the works of the Pythagoreans, Euclid, Archimedes, Apollonius, Hero, Diophantus, Hipparchus, Ptolemy, Pappus, and Proclus, among others, but there is no space to describe anything in detail. Although we learn that there is no "reliable date or place of work" for Euclid and that even "his traditional association with Alexandria has little basis in ancient documents," there is too little on the *Elements* for a reader to gain any understanding of the influence this work has had for 2300 years. Archimedes and Apollonius get a bit fuller treatment, and some attention is paid to the Greek interest in geometrical constructions for solving problems, but we learn very little of Ptolemy's trigonometrical methods or of Diophantus's numerous methods for solving indeterminate equations.

Thus, the traditional topics of ancient mathematics, which in textbooks on the history of mathematics take up 25–35% of the pages, and in Kline's work 15%, are given only about 4% of the space in the *Encyclopedia.* Grattan-Guinness has evidently decided that these topics are not so important, but it seems to me that it is difficult to understand much of the history of more modern mathematics without a firm grounding in ancient material. After all, all of the important mathematicians of the 16th, 17th, and 18th centuries received their initial mathematical education in the Greek classics. And the Greeks themselves learned something—although we do not know exactly what—from their Babylonian and Egyptian predecessors. The reader looking for a good introduction to Egyptian and Babylonian mathematics would do better in one of the standard texts, while Kline's work is certainly superior for Greek mathematics, as is the *DSB,* at least for individual Greek mathematicians. In addition, it would be very difficult for a teacher to find enough examples in the *Encyclopedia* to use in the classroom. On the other hand, Grattan-Guinness's authors do provide plenty of up-to-date references to put the reader on the right track to find out more.

What can you learn about the mathematics of medieval times, between the end of the Greek era and the beginning of the European Renaissance? The *Encyclopedia,* like most texts, contains separate sections on the mathematics of China, India, and the Islamic world. Jean-Claude Martzloff describes the background for and the basic achievements of Chinese mathematics, while Takao Hayashi does the same for India. Jan Hogendijk and David King split the duties for Islam, with the former discussing pure mathematics and the latter aspects of mathematics applied to religious ritual. Although the chapters are too brief, the authors try diligently to include as much as possible. Thus we learn that the decimal place-value system goes back in India probably to the third century A.D. and that the Pascal triangle can be found there in the 10th century. We get details about the decline of indigenous Chinese mathematics after the 14th century. And the chapters on Islam present most of the high points of Islamic mathematics. Unfortunately, because of the brevity, some of the mathematical achievements of these civilizations are slighted and virtually all mathematical details are left out. In particular, the Islamic chapters barely mention the detailed work on combinatorics accomplished in the Maghreb (although that work is considered briefly in a later section on combinatorics itself) and devote only one sentence to the extensive efforts made in Islam to prove Euclid's parallel postulate.

We do get a few bonuses not generally found elsewhere. Tamotsu Murata discusses *wasan*, the indigenous Japanese mathematics, and George Joseph details Tibetan work on mathematical astronomy. We also learn from Claudia Zaslavsky about mathematical practices in Africa and from Yong Woon Kim about certain aspects of mathematics in Korea. And finally, Tzvi Langermann gives a rapid survey of Jewish contributions to mathematics during the medieval era.

European mathematics in the Middle Ages forms part of several different sections in the *Encyclopedia.* The Euclidean tradition is treated by Jürgen Schönbeck and the very extensive tradition of practical geometry, partly based on the surviving fragments of Roman agrimensure, gets even more space in a wonderful article by Hervé l'Huillier. Eberhard Knobloch teaches us about some aspects of medieval technology, while George Molland covers medieval mechanics and kinematics. We also get two excellent articles, by Gillian Evans and by Molland, on the teaching of mathematics and the philosophical context of mathematics during that time period.

To summarize, you would learn bits and pieces about the nature of medieval mathematics all around the world by perusing the *Encyclopedia,* but, except in the case of Europe, would only gain some general ideas about the mathematics itself. You would be aware of some of the problems posed in that time period, but only have a vague notion of the solutions. For this time period, Kline's work would be no better, and the *DSB* would be difficult to use; some of the standard texts, however, would provide much more in the way of material usable in the classroom. But again, as in the earlier time period, the extensive bibliographies in the *Encyclopedia* would help the reader find out more.

What could we learn about mathematics in Europe during the Renaissance? The teacher looking for classroom material, as well as the general reader, could learn quite a bit. For example, the "coss" tradition in algebra in Italy, France, and primarily Germany, is given detailed consideration by Karin Reich, while the reader will learn exactly how Napier and Briggs computed logarithms in the article by Wolfgang Kaunzner. Kirsti Andersen then discusses the numerous approaches during the period 1635–1665 to finding tangents and quadratures, including the work of Fermat, Pascal, Descartes, and Roberval. But if we ask about the work of Newton and Leibniz in putting together many of the previous ideas and techniques into the wonderful instrument called the calculus, then we are again disappointed. Presumably because they contributed in so many ways to mathematics, the ideas of these two geniuses are scattered throughout the *Encyclopedia.* One can understand perhaps that ideas of physics would be separated from ideas of mathematics. But even work which seems to fit together both topically and chronologically is separated. Thus, Newton's work on power series, which was a central part of his calculus, is treated (by Lenore Feigenbaum) 200 pages after Niccolò Guicciardini's brief summary of Newton's introduction of the fluxion and the fluent, while his work on the binomial theorem is in a third article, this one by Michael Pensivy. Thus, if you want an introduction to Newton's ideas on calculus and their relationship to the ideas of others of the time, you would need to scan these and several other articles to be able to put together any decent picture. A similar task would be necessary for the student to learn about Leibniz. Most of the standard textbooks present a much stronger introduction to the history of calculus in general and the work of Newton and Leibniz in particular. In fact, there is far more in these texts that a calculus teacher could use in teaching the subject than will be found in the *Encyclopedia.*

How about the development of analysis after the time of Newton and Leibniz in the *Encyclopedia*? Detlef Laugwitz gives a whirlwind tour of 140 years of real analysis from Cauchy to Abraham Robinson in 13 pages, with the last several being devoted to reworking some of Cauchy's proofs in the new language of nonstandard analysis. Umberto Bottazzini then takes the same amount of space to cover only about 70 years of complex analysis from Cauchy to Weierstrass, so you would not have quite the same rushed feeling. But if you blinked, you would miss entirely Euler's contributions to shaping the subject. His three major texts of the mid-18th century are given perhaps four pages, scattered among three different articles. An equally sparse treatment of ordinary differential equations by Christian Gilain skips too many of the elementary ideas of the subject to be helpful to the beginner. On the other hand, Kenneth Gross's treatment of harmonic analysis is quite long and detailed, as is Jesper Lützen's piece on partial differential equations and James J. Cross's article on potential theory.

Although the *Encyclopedia* does not in general succeed in giving strong enough introductions to mathematical topics prior to the 19th century, it gets much better once that time is reached. For almost any mathematical topic you can name from that time period, there will be an article—or perhaps more than one. Of course, as in the earlier sections, the articles vary in quality and some might better have been left out altogether. In certain cases, although not as often as in the earlier sections, Kline and the standard texts have sections superior in quality.

The part of the *Encyclopedia* devoted to logic, set theory, and foundations contains numerous excellent articles. If you want to learn about the paradoxes of set theory, Alejandro Garciadiego enlightens you. If you need to understand the relationship between set theory and logic, you can consult Gregory Moore. Or if you want a brief but to the point study of constructivism, you will find it in the work of Michael Detlefsen. Similarly, Doron Swade, Martin Campbell-Kelly, and Steve Russ give very nice surveys on the history of calculating machines and computers.

In algebra, we have an excellent, but brief, survey of the theory of equations by Laura Toti Rigatelli, a wonderful discussion of the history of continued fractions by David Fowler, and a marvelous overview of the development of the notions of groups, rings, and fields by Hans Wussing and Walter Purkert, with a long bibliography to aid the reader in pursuing ideas further. Similarly, Ivor Grattan-Guinness and Walter Ledermann outline the history of matrix theory, while Helena Pycior gives a very nice summary of the philosophy of algebra. But there are weak articles here as well. It would be very difficult for a student to learn much about invariant theory from Tony Crilly's survey and, although Günther Frei tries hard, it is simply impossible to give even a decent sketch of the history of number theory in 22 pages. Linear optimization and operations research are certainly subjects of current interest, but you would not gain much understanding of either the problems being considered or the solutions fround from reading the two articles here.

The part of the *Encyclopedia* devoted to geometry is less successful, although it contains a splendid article by Joan Richards on the philosophy of geometry to 1900. We are, however, tantalized by her concluding sentence, "the 20th century has opened a whole new chapter in its development"—and that chapter is not presented here at all. Most of the other articles in this part are simply too short to enable us to understand the topic under consideration. Thus Jeremy Gray's treatment of analytic geometry fails to give any analysis of Descartes's methods, and his chapter on curves barely scratches the surface. His article on Euclidean and non-Euclidean geometry is better, but fails to discuss the Islamic contributions or their relationship to the work of Saccheri and Lambert and leaves us hanging at the end with a cryptic sentence on the work of William Thurston. And his work on algebraic geometry does not give any examples to allow a student to visualize some of the general theorems. A similar criticism applies to Erhard Scholz's article on topology. One also wonders why there are separate articles on line geometry and on invariance of dimension, two topics which could easily have been included as parts of more comprehensive articles. As it stands, one does not understand how they are connected to the more mainstream ideas. This part of the encyclopedia also contains Jeremy Gray's brief discussion of the development of the notion of a vector space, including Peano's axiomatization of the concept, but fails to note that it took many decades for this axiomatization to be accepted. Finally, Robin Wilson and Keith Lloyd conclude this part with a good survey of various concepts of combinatorics, but one which misses some of the significant medieval contributions.

Grattan-Guinness has criticized other surveys of the history of mathematics for leaving out large segments of what today we would call "applied mathematics" but which in the 18th and 19th centuries were significant parts of the mathematical enterprise. Thus, in this work, he has made sure that these are given their proper importance. In fact, 300 pages are devoted to mechanics, physics, and mechanical and electrical engineering. This part of the encyclopedia contains much material not found in the competition, although, of course, many students who have only studied "pure" mathematics will find these articles tough going. Nevertheless, if you want to find out about theories of elasticity or the three-body problem, geodesy or tides, shipbuilding or thermodynamics, acoustics or statistical mechanics or mathematical theories of electricity and magnetism, here is the place to start. You can even get a brief introduction to the uses of mathematics in chemistry and biology.

If your interests are more on how mathematics is used in the social sciences, Grattan-Guinness again does not disappoint, with 140 pages devoted to probability and statistics, including applications to numerous areas. There are wonderful articles by Eberhard Knobloch on combinatorial probability and by Glenn Shafer on the early development of mathematical probability. And there is a fascinating article on the beginnings of actuarial mathematics by Christopher Lewin, although this would have been helped by more definitions of unfamiliar actuarial terms and by a few examples. If you wish to learn about how probability and statistics have been applied to psychology or genetics or agronomy or medicine, you will find excellent introductions to the fields with numerous references. And there is also a superb philosophical summary by Theodore Porter of the uses of probability and statistics in the social sciences. But one does wonder why there is a separate article on Russian probability and statistics as well as one on the theory of errors which overlaps considerably with earlier articles.

The *Encyclopedia* contains another part which is not found in any of the competitive works, a collection of articles detailing higher education and institutions in various countries which have been significant in mathematics. Grattan-Guinness himself writes about France, Scandinavia, Britain, and Russia, the latter along with Roger Cooke. We also have Gert Schubring discussing Germany, but stopping before the Nazi period, Christa Binder dealing with Austria and Hungary, but unfortunately not telling us why Hungary grew to have such a strong mathematical tradition, and Jan van Maanen writing about The Netherlands, with a nice treatment of the golden age. Karen Parshall and David Rowe also give us a detailed summary of their important new book on American mathematics in their own article.

The final part of the *Encyclopedia* is devoted to a potpourri of topics, under the general heading of ''Mathematics and Culture.'' The best of the articles is David Singmaster's marvelous survey of the history of recreational mathematics, but there are also interesting pieces by Marcia and Robert Ascher on ethnomathematics, Roger Herz-Fischler on the golden number, Joseph Malkevitch on tilings, Helena Pycior on mathematics and prose literature, and John Fauvel on mathematics and poetry, among others. There are even articles on mathematics on stamps by Hans Wussing and mathematical monuments around the world by David Singmaster. Grattan-Guinness concludes this part with a survey of the history of the history of mathematics, while the *Encyclopedia* as a whole concludes with bibliographical, biographical, and chronological tables as well as a detailed index.

In looking through the *Encyclopedia*, I often found myself wishing that the articles were not so short. Many authors seemed to want to dig deeply into a particular area, but were thwarted by a page limitation, so they could only give brief overviews instead of detailed discussions. It would seem that a long article on the development of the calculus would have been more useful to readers than the many brief references to the work of Newton, Leibniz, and their predecessors and successors scattered throughout the *Encyclopedia.* A long discussion of the history of equation solving would make more sense and prove more useful in education than the several all too brief articles which touch on this subject. And although the editor himself provides a brief summary article on trigonometry which attempts to tie together the mentions of the subject elsewhere, a single longer article on the subject would be more helpful for a teacher planning to use this history in class. In general, the brevity of most of the articles prevents them from having enough material to be of direct use in a mathematics classroom.

In the end, I have very mixed feelings about the *Encyclopedia.* I am certainly impressed with the experts that Grattan-Guinness assembled. I had looked forward to learning much from them, and, in many cases, I will. In fact, whenever I have a question in the history of mathematics, I will begin by consulting the *Encyclopedia*. But I had also hoped that the *Encyclopedia* would be useful when students ask me about the history of a particular topic. Here, except in the case of topics in applied mathematics, the *Encyclopedia* will often fail to provide a strong enough introduction, and I will recommend either Kline's work or one of the history texts. Perhaps I expected too much from this (or any) encyclopedia; but I still think there would be a place for an extensive work, compiled by a team of experts, which contained fewer but meatier articles on the major topics in the history of mathematics.

ARTICLE NO. HM972161

From Five Fingers to Infinity: A Journey through the History of Mathematics. Edited by Frank J. Swetz, Chicago and La Salle, IL (Open Court). 1994. 770 pp.

Reviewed by Robin Wilson

Faculty of Mathematics, The Open University, Milton Keynes MK7 6AA, England

This is a massive book, covering an impressive variety of material. Unlike traditional texts on the history of mathematics, it is a compilation of over one hundred attractive articles on topics ranging from the Peruvian quipu to Fermat's last theorem, from Ptolemy's trigonometry to Cayley's matrix theory, and from Bhaskara's Lilavati to the four color problem. The articles are taken from a number of sources and are mostly short and self-contained, chosen to be read individually or in sequence.

The material is organized into eight parts, or chapters:

I. Why the history of mathematics?
II. In the beginning
III. Human impact and the societal structuring of mathematics
IV. European mathematics during the "dark ages"
V. Non-Western mathematics
VI. The revitalization of Western mathematics
VII. Mathematical responses to a mechanistic world outlook
VIII. The search for certainty

Yet the details of its historical development remain obscure to all but a few specialists. The two-volume Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences recovers this mathematical heritage, bringing together many of the world's leading historians of mathematics to examine the history and philosophy of the mathematical sciences in a cultural context, tracing their evolution from ancient times to the twentieth century.In 176 concise articles divided into twelve parts, contributors describe.Â This indispensable reference work demonstrates the continuing importance of mathematics and its use in physics, astronomy, engineering, computer science, philosophy, and the social sciences.