The future of paraconsistent logic

Jean-Yves Béziau

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1 Logic of the future or logic without future ?

Stefan Zweig has written a book called *Brazil, country of the future*, it does not seem that his prophecy has already been fulfilled and there is a typical Brazilian joke saying that Brazil is the country of the future and will stay so forever.

Maybe one can say a similar thing of paraconsistent logic. After its creation, nearly 50 years were necessary to organize a world congress on paraconsistency and there is still no textbook dedicated to it. For comparison, linear logic, only ten years after its creation, is already studied worldwide, and many linear congresses have been organized. But it is difficult to know what will happen. Maybe paraconsistent logic will last and linear logic is just a fashionable logic that nobody will remember in ten years. Or maybe paraconsistent logic will stay a controversial curiosity and linear logic will turn into the official logic, replacing classical logic, ruling our computers, wifes and money.

Or maybe paraconsistent logic and linear logic are only variations on the same theme as classical logic. Maybe in the future there will be more radical changes, a logical revolution similar to the Fregean one. A new conception of logic will take place relatively to which classical logic will look like syllogistic looks today relatively to classical logic, and paraconsistent logic and linear logic will look like the numerous variations and alleged improvements of syllogistic that were proposed during the 2000 years in-between Aristotle and Frege, that nobody, apart from historians and philologists, is interesting in today.

2 Paraconsistent problems

2.1 The problem of definition of paraconsistent logic

Paraconsistent logic?

Roughly speaking, a paraconsistent logic is a logic rejecting the principle of non contradiction (PNC). The question is whether a negation not obeying PNC is still a negation. For some people like Quine [82], and more recently Slater [87], the answer to this question is a big NO because negation must *by definition* obey PNC. But if one thinks two minutes about the problem, he will see that it is not that simple.

What is the definition of negation? The implicit claim of Quine and Slater is that there is only one negation, which is perfectly and precisely described by classical formal logic. However, on the one hand it is difficult to argue that this classical negation gives an account of the use of natural language's negation and therefore such definition appears to be not very descriptive but rather strongly normative, not to say dictatorial. On the other hand, even in mathematical logic, since many years people call "negations" various connectives close to, but different to classical negation.

People may argue that PNC is one fundamental property of negation, and that it makes no sense to still use the word "negation" to denote a connective which does not enjoy this property. But the principle of excluded middle is also a fundamental property of negation, a kind of dual of PNC, and intuitionistic negation which does not enjoy this property is nevertheless called a negation. So why one should not be allowed to call a negation an operator which may appear as a dual (cf. Section 3.4.) of intuitionistic negation?

The question of whether a paraconsistent negation can rightly be called a negation is still an important open problem which has two sides: 1) On the mathematical side we must investigate which are the properties of negation which are compatible with the rejection of PNC. 2)On the philosophical side, we must see if these properties are enough to characterize the idea of negation. There are still a lot of work to do on both sides. On the mathematical side the question is not so simple because properties of negation are of different types and levels and they depend on the framework in which we are working and there are several possible frameworks (standard Tarskian consequence operator, substructural sequent calculus, etc.). What is needed here to properly conduct such investigations is a general theory of logic, a universal logic, which will play the same role as universal algebra plays in algebra (cf [18], [21], [31]). On the philosophical side, there also various possible ideas or theories of negation which are not compatible and which belong to different contexts.

"Paraconsistent logic"?

As it is known, the expression "paraconsistent logic" was proposed in 1976 by the Peruvian philosopher Miró Quesada as a possible solution to a request by da Costa. Very quickly the name became widespread and this name even play a decisive role in the propagation of its denotation (cf. [43], [49]).

However there are many people who don't like it and have tried to replace it, without success, by something else : dialethic logic, antinomic logic, parainconsistent logic, logic of contradiction, teratologic, transconsistent logic, etc...

The Greek prefix "para" has mainly three meanings:

- (1) "against" as in the expression "paradox" (which means in Greek, against the common sense),

- (2) "beyond" as in the expression "paranormal",

- (3) "very similar" ("connected", "nearby") as in the expression "paramilitar".

This perfix is therefore itslef contradictory in the sense that it has different incompatible meanings. Some people reject the expression "paraconsistent logic" because they interpret it according to the the third meaning of "para" and they think that such interpretation does not match the revolutionary purpose of paraconsistent logic which is to take us not nearby but beyond consistency. The people who suggest the expression "parainconsistent logic" think that paraconsistent logic is beyond, or nearby inconsistency, not consistency.

What is less known is that the prefix "para" exists in other languages with other meanings. In Puppy-Guarana, one of the many disappeared languages of the indians of South-America, it means "out of". Although Puppy-Guarana is not anymore spoken, many proper names in Brazil are from this language, like the name of Rio's beach "Ipanema", or like the name of the city "Parati" which means "out of you", or like the name of the State where da Costa was born, "Parana", which means "out of Na", Na being the name of a famous native Goddess (cf. [36]).

The expression "paraconsistent logic" is nowadays well-established and it will make no sense to change it. It can be interpreted in many different ways which correspond to the many different views on a logic which permits to reason in presence of contradictions. Nevertheless one may want to have further names to name these views. Interpreting the prefix "para" in its native sense, along the line which leads from left to right, from the conformist comfortably sitted on the firm dogma of consistency to the nihilist standing up, holding high the burning flag of inconsistenty, we have the following picture:

consistent, neoconsistent, transconsistent, anticonsistent, paraconsistent, parainconsistent, antiinconsistent, transinconsistent, neoinconsistant, inconsistent.

2.2 Philosophical problems

Some people believe in True Contradictions and for that reason defend the existence of paraconsistent logic. Other people believe in God and for that reason defend the existence of the Church. In both cases the difficulty is to know what is exactly the object of their belief. And to deduce the existence of True Contradictions from paraconsistent logic seems to be possible only by a reasoning similar to the one with which we will deduce the existence of God from the existence of the Church, reasoning common among people from the Church, but rare among philosophers, Kant being the exception that proves the rule.

In some sense we are surrounded by contradictions. But maybe these contradictions are just apparent or maybe they really exist but are things that must be avoided. In any case it seems that paraconsistent logic is essential, because even if contradictions are only apparent, perhaps we cannot escape from the world of appearances, we have to stay in Plato's cave dealing with contradictions, and even if we can or must avoid them, perhaps paraconsistent logic is a smoother way than classical logic to do so.

Some people think that paraconsistent logic is essential because contradictions are essential. They think that there are true contradictions that cannot be avoided or more than this : contradiction is the essence of reality.

Clearly there is a fight over contradictions. Some people are against them, they think that contradictions entail confusion and that they must be banished from rational discourse, from political rhetoric and scientific method. On the other hand there are people who think that contradictions are everywhere, in the opposition between day and night, good an evil, truth and falsity and that these couples are the very generating forces of reality. That life results from the copulation between God and Satan, that there is no happiness without suffering, no truth without falsity.

People who love contradictions think that the straigth distinction between falsity and truth is too artificial, people who hate them think that confusion between truth and falsity must be avoided because it favours falsity rather than pure white truth. On both sides people are more interested to say what they think contradiction is rather than to try to understand what it is, i.e. on both sides there are few true philosophers. This is confirmed by what people write on the subject which is a blend with a low philosophical density.

Let us take just one example. Some people say that the liar paradox proves that a natural language such as English is contradictory and that this proves the existence of true contradictions. But I can write "It is raining over Paris and it is not raining over Paris", this will not entail that 2+3=7 or that God is Satan. And even if I say it, the earth will not explode. We must confuse language neither with reality nor with reasoning.

People may argue that if it does not appear as a contradiction, it is because we are taking it at a poetical level. Taking it as its right level, the assertorical (vs. poetical) level, together with a correspondance theory of truth, it really entails that the earth is a big apple pie, and since it does not seem so, one has to admit that it is not possible that it is raining over Paris and it is not raining over Paris, or one has to use paraconsistent logic. But why should we take it as the assertorical level? Maybe the liar paradox is just a product of the poetical power of natural language.

The future of paraconsistent logic will depend on a right analysis of the various different aspects of contradiction. People saying that there are contradictions everywhere, without any rational support, and that paraconsistent logic is therefore our salvation, are similar to people alerting about a terrible disease in order to sell a wonderful drug especially made to cure this disease. But maybe there is no disease and the drug is a placebo. A group of people can get power by saying that they can control dangerous creatures that are the product of their imagination. This remembers the story of religions frightening people with devils in order to convert them to their Churches.

By contrast to such kind of religiously oriented philosophical approaches to paraconsistency, mathematical logic can play a fundamental role in the philosophical future of paraconsistent logic, positively by constructing mathematical systems in which there are interesting paraconsistent negations or negatively by showing that there are no such systems.

2.3 Mathematical problems

<u>Looking for a "good" paraconsistent logic</u>

One thing is the idea of paraconsistent logic, another thing is a mathematical system corresponding to this idea.

Obviously, Vasiliev had the idea of paraconsistent logic, but he didn't built a system of paraconsistent logic (cf. [7], [12]). Since then many paraconsistent systems have been proposed, but all of them have some basic defects, so that it is not clear at all if paraconsistent logic really exists in the sense that there exists a real paraconsistent logic. But what is a "real" paraconsistent logic ?

We can propose the following conditions:

- Negative criterium (rejection of the ex-falso sequitur quod libet)
- Positive criterium (strong properties which allow to speak about a negation)
- Good intuition
- Nice mathematical features.

There are several properties a negation can have, the question is to know how one can choose a set of desired properties and build a system having those properties and only those. Mathematics can put an end to some dreams, showing that it is not possible to build such or such system.

A systematic mathematical study of negation can show which properties are compatible with each other and which collection of properties can be unified in a paraconsistent sysem. Such systematic study has not yet been carry out very far. Most paraconsistentists have either a constructivist attitude, trying to build their own better edifice in the paraconsistent town, or either a deconstructivist attitude, trying to destroy other systems finding malignant hidden defects. It is a pity because several general results about paraconsistent negations are more significative that such or such ill-constructed system.

Unfortunately these general results are not very well-known, not because they are difficult to understand but maybe because they are negative. For example it is very easy to show (cf. [19]) that all the forms of contraposition and reductio ad absurdum are incompatible with the idea of paraconsistency, if we take these principles relatively to the consequence relation (e.g. if $\neg a \vdash b$ and $\neg a \vdash \neg b$, then $\vdash a$; if $\neg a \vdash \neg b$ then $b \vdash a$). Weak forms of these principles hold for negations such as intuitionitic and minimal negations. In the case of paraconsistent logic, such principles are valid only for a kind of implications which have very weak properties.

Paraconsistency and self-extensionality

Anyway there are some partial results related to such and such system or of a wider width. It seems that the main problem is connected to the compatibility between the replacement theorem and basic properties of negation. Following Wójcicki's terminology (cf. [99]), let us call self-extensional a logic in which the replacement theorem holds. The logic C1 is not self-extensional (see [53], [54]). Mortensen [70] has shown moreover that there are no non trivial congruences relations definable in C1. However it is possible to develop extensions of C1 in which there are some non trivial congruence relations (see [71], [50]. But Urbas has shown that C1 admits no non trivial self-extensional extensions other than classical logic [93].

We don't think that self-extensionality is necessarily a mathematical defect for a logic (cf. [24]). But from the philosophical point of view there must be an intuition supporting the non self-extensional behaviour of a negation.

In C1, $a \wedge b$ is logically equivalent to $b \wedge a$, but $\neg(a \wedge b)$ and $\neg(b \wedge a)$ are not logically equivalent. In LP, $a \vee \neg a$ is logically equivalent to $b \vee \neg b$ but $\neg(a \vee \neg a)$ and $\neg(b \vee \neg b)$ are not logically equivalent. And in both cases there are no intuitive justifications of these "abnormalities".

One has to find a non self-extensional paraconsistent logic in which such kind of abnormalities do not appear, or to work with self-extensional paraconsistent logics. Let us note the intrinsic limitations of negation in a self-extensional paraconsistent logic. It has been shown (see [26]) that such a negation cannot obey simultaneously the double negation law and the principle of non contradiction in the form $\neg(a \wedge \neg a)$. Consequently a self-extensional logic in which all De Morgan laws hold and in which moreover holds the principle of excluded middle cannot be paraconsistent.

Examples of self-extensional paraconsistent logics are the logic P1 [84], the logic LDJ [95] and the logic Z [32]. P1 is an *atomical paraconsistent logic* in the sense that only atomic formulas may have a paraconsistent behaviour. LDJ is a *classical paraconsistent logic* (cf. [90]) in the sense that it admits all theorems of classical logic. Atomical paraconsistent logic and classical paraconsistent logic are both difficult to defend philosophically. But even if one can find serious philosophical arguments in favour of atomical paraconsistency or classical paraconsistency, the weak point of these logics is maybe that they delimit big logical domains for paraconsistency but the behaviour of paraconsistent negations in these regions is not strong enough.

The problem of a "big" paraconsistent logic

Many non classical logics are, at the propositional level, funny toys which work quite good, but when one wants to extend them to higher levels to get a real logic that would enable one to do mathematics or other more sophisticated reasonings, sometimes dramatic troubles appear. This is for example the case of modal logic. In the case of paraconsistent logic, it is difficult to have an opinion, since the problems are not even solved yet at the propositional level.

If one extends a non self-extensional propositional logic like C1, one will meet at the higher levels the same problems that were found at the lower level, that can get worst or not, and new problems may also appear. In the case of C1, in order to avoid further new problems one has to be cautious. In the usual construction of the language of first-order logic, two formulas like $\forall x\phi x$ and $\forall y\phi y$ are different but they are logically equivalent. In the non self-extensional first-order version of C1, in order to "identify" these two formulas, one as to put an ad hoc extra-axiom [42], or to modify the construction of the language using for example Bourbaki's square [22] ; in this latter case these two formulas are logically equivalent because they are one and the same formula. To deal with non self-extensionality one has also to modify the notion of isomorphism in order to preserve the concept of identity (cf. [23]). Isomorphism must be *defined* as preserving the structure not only at the atomic level (aRb iff $\iota aR\iota b$) but also at all levels ($\phi(a_1, ..., a_n)$ iff $\phi(\iota a_1, ..., \iota a_n)$). Further problems appear with definitions [48], if for example one wants to introduce in set theory constants such as $\emptyset$ or ω in the formal language. All these problems are related to non self-extensionality and therefore appear in all non self-extensional logics, be it C1, or J3 or LP. In some cases, where there are some congruent connectives within the language (C1+ or Buchsbaum's logics [37]), it is a little simpler but the basic problems persist.

What happens in the case of a self-extensional paraconsistent logic? In the case of the paraconsistent logic Z, one is confronted for its possible extension to higher levels to the same problems that appear in modal logic, these problems maybe regarded as negligible side-effects if we don't focus on the modal aspects of this logic. It seems difficult to use a classical paraconsistent logic, like the first-order version of LDJ, for developing paraconsistent mathematics, since mathematics is rather about theorems and this kind of paraconsistent logic have the same theorems as classical logic. The situation of atomical paraconsistent logic is not better since contradictions when they pop in mathematics rather pop at a high level of complexity. Even a very simple contradiction like Russell's paradox is not an atomical contradiction. Maybe a paraconsistent logic suitable for mathematics should be a *complexical paraconsistent logic* where formulas can have a paraconsistent behaviour only above a given degree of complexity (see [33]).

Paraconsistent set theory and Curry-Moh Shaw Kwey's paradox

The Curry-Moh Shaw Kwey's paradox shows how to derive anything from the unrestricted axiom of abstraction with the help of very few additional logical principles and in particular no principles concerning explicitly negation. Therefore paraconsistent logic is not really a possible alternative, for escaping Russell's paradox, to ZF or other classical set theories based on the restriction of the axiom of abstraction.

However paraconsistent logic permits to deal with set theory with a restricted axiom of abstraction together with a Russellian set or other paradoxical sets (see [46], [52]), [57]). To be of interest the underlying negation must be strong enough to warrant that such sets are really paradoxical and not just only apparently or poetical contradictory sets. Paraconsistent logic has not yet appeared as useful for some new foundations of mathematics but the future will maybe bring some surprises through some more audacious tentatives such as logics in which a formula can be identical to its negation (cf. [67]).

3 Relations between paraconsistent logic and other logics

It seems to us that the future of paraconsistent logic will depend on its relations with other logics and in particular we don't share the belief of paraconsitent Darwinians who have an evolutionary view of logic according to which only paraconsistent logic will survive in the future because it is the strongest and best logic. Neither we share the view of classical Darwinians and relevant creationists. We think that logic is in an endless motion, that there is no final super-logic, that the nature of this motion can be studied within a general theory of logics (cf. Universal logic), that a logic can properly be understood only by means of a comparative study investigating its relationships with other logics.

3.1 Fuzzy logic and many-valued logic

Let us try to explain the relations between logics in terms of colours: classical bivalent logic is black and white, a many-valued logic is so much multicoloured as it is many-valued, fuzzy-logic is densely multicoloured including all the colours between purple and pink, yellow and green, black and white. And so what would be the colour of paraconsistent logic? Red, like the fire of a revolutianory logic? Deep blue, like the artificial intelligence of the computers' star of the third millenium? Unfortunately it seems that its colour is not so poetical, maybe just black and white with a pinch of white in the black and a pinch of black in the white, like the Tao's symbol. Of course it is possible to mix paraconsistent logic with multicoloured many-valued logic and to get a pinch of red into a sea of blue and a pinch of yellow into a black see.

From the technical point of view, the relationships are more prosaic. One can for example use many-valued matrices to try to define a paraconsistent negation, but the success is not guaranteed. In the course of history, Asenjo [5], D'Ottaviano/da Costa [58] and Priest [76] have presented paraconsistent negations constructed with many-valued matrices, but none of these logics has a satisfactory behaviour. In particular they all are non self-extensional (cf. [26]). Many-valued matrices have also been used as a semantics for atomical paraconsistent logics like Sette's logic [84] and Puga/da Costa's formulation of Vasiliev's logic [80]. More recently people like Karpenko [64], Avron [6], Tuziak [92] or Almeida [2] have investigated in a more general way what can be done with many-valued matrices in the field of paraconsistent logic. Furthermore the present author [16] has proposed to extend the traditional conception of many-valuedness based on matrices to non truth-functional semantics and has presented such semantics for the paraconsistent logic C1 and other ones.

At first sight, many-valuedness seems an attractive idea for paraconsistent semantics. But it may be just an illusion. One has to be careful with the mixture between a binary division in the language (a and $\neg a$) and multiple values in the semantics, and more generally with the bendy philosophy of many-valuedness, according to which there are in some sense many-values, but in another sense the principle of bivalence is preserved with the binary division between designated and non-designated values (see [51], [25]). Many-valued matrices are for sure a useful mathematical tool for paraconsistent logic as it is for many logics such as classical logic, when it is used for example to prove independence of axioms (cf. [3]). But it is not so clear that they are philosophically relevant for paraconsistency. History has shown that in the case of modal logic, many-valued matrices do not work, although at first sight it seems that there is a connection between possibility and a third value. Maybe the same is true of paraconsistent logic in the sense that a good paraconsistent logic cannot be characterized by a finite matrix. Maybe, like for modal logics, Kripke semantics is better adapted (see Section 3.3), or maybe the perfect paraconsistent semantics must be constructed using new technics such as Carnielli's possible-translation semantics (cf. [40],

[2]) or Buchsbaum's min-max semantics [37].

3.2 Relevant logic

For many relevantists classical logic is totally wrong and must be rejected because it is not "relevant", in the sense that you can deduce from a set of assumptions a conclusion which says something which has nothing to do with what the assumptions are about. The ex-falso sequitur quodlibet is not a "relevant" deduction. For the relevantists this is one more example which shows how much classical logic is wrong and they think that paraconsistentists are right to reject it, but they also think that they must not stop there, they must go on and reject all other non relevant deductions.

Relevantists like to say that paraconsistent logic is part of relevant logic and paraconsistentists like to say the opposite. In fact a relevant logic can be paraconsistent in no significant way, and this is the case of the usual relevant logics. This illustrates perfectly the fact that the rejection of the ex-falso sequitur quodlibet is not enough to define paraconsistency. And it is also very clear that it is possible to develop paraconsistent logics which are not relevant : typical examples are paraconsistent logics which are finitely trivializable or which admit a classical negation.

Maybe the relevantists are right and in the future our reasonings will be completely meaningfully mechanized in such a way that it will not be anymore possible to take a rabbit out of a hat, the magic of the ex-falso fading away. But this does not necessarily imply that we will live in a world of contradictions. Or maybe the paraconsistentists are right, we will live in a world full of meaningful non exploding contradictions, but allowing also some magic touches of purely formal deduction. Or maybe the logic of the future will be the daughter of relevant logic and paraconsistent logic, a logic of contradictory meanings.

3.3 Modal and intensional logics

In the same way that we can extend classical logic into various modal logics (S4, S5, B52, F242, etc) by adding the modal operators of possibility and necessity, we can extend the various modal logics and neo-modal logics (tense, epistemic, doxatic, deontic, faith, belief logics, etc) into paraconsistent logics by adding paraconsistent negations.

There are many intuitive motivations, stronger in this context than in pure extensional logic. For example, contradictory beliefs is a natural concept in our world in which rockets are sent out of the solar system diffusing music to make the universe beat at the rhythm of human craziness, and at the same time many people still think that earth is as flat as a CD manufactured by a ingenious god or a "malin génie" to play the eternal song of human sorrow.

But to deal with contradictory beliefs (fears, obligations, etc.) do we really need a paraconsistent logic? What we want is to be allowed to believe in God and Satan without admitting anything such as the existence of green creatures on Mars. But maybe this can be done without rejecting the ex-falso sequitur quodlibet. To believe in Satan does not necessarily mean not to believe in God, it can mean to believe in a anti-God. Formally speaking (we use the symbol "$\circ$" for belief representation), we may have a logic of belief in which $\circ a, \circ\neg a \nvdash b$ but in which $\circ a, \neg \circ a \vdash b$ and more generally $a, \neg a \vdash b$. So this logic may not be, strictly speaking, a paraconsistent logic. The question to know if we can deal with contradictory beliefs, in this sense, without entering the realm of paraconsistency has been put forward by J.Wainer [98]. Anyway, in case of a positive answer to this question, it seems to us that instead of saying that there are logics of contradictory beliefs which are not paraconsistent, it would be better to modify the definition of paraconsistency in order to include this kind of logics within the realm of paraconsistency. More generally, given an "intensional" operator $\heartsuit$, one can say that a logic in which $\heartsuit a, \heartsuit\neg a \nvdash b$ but in which the ex-falso holds is a paraconsistent logic, or to be more precise is a *heartian paraconsistent logic*.

In fact, combinations of intensional operators such as standard modalities with standard extensional operators such as classical negation can lead to paraconsistent negations. That what has been recently pointed out by the author [30]. For example, in S5, $\diamond\neg$ is a paraconsistent negation with interesting properties. From this point of view, the relationships between modal logics and paraconsistent logics seem to be a very promising field of investigation for the future: on the one hand we can produce interesting new paraconsistent negations using intuitions and techniques of modal logics (in particular Kripke models), on the other hand we can generate intensional operators with paraconsistent negations.

3.4 Intuitionistic logic, paracomplete logics and paranormal logics

Intuitionistic logic appears as a dual of a particular paraconsistent logic. Reverse intuitonistic logic, put his head down and his foots up, his foots will look like a head and his head like some foots, and you will get another logic, a paraconsistent logic. But there are several ways to reverse intuitonistic logic, it all depends on how you see it, you define it. If you look at it through a sequent calculus, you will reverse this vision by admitting only one formula on the left and you will get something [95] which is not necessarily the same as the reverse vision of another vision, such as a the dual of a Heyting algebra [85], the dual of a Kripke semantics, or the dual of a topos (cf. [66], [63], [97]).

Moreover there are many other paraconsistent logics than all the reverse visions of intuitionistic logic. And all the reverses of paraconsistent logic are many more than all the possible visions of intuitionistic logic, and form the rich field of *paracomplete logics* [68]. Each paraconsistent logic has a paracomplete dual and each paracomplete logic has a paraconsistent dual. The duality between paraconsistency and paracompleteness can be worked out with any semantical idea, informal interpretation, etc. For example from the viewpoint of game semantics, following the idea of T.Pequeno, paraconsistency corresponds to a game where both players can win, by contrast to paracompletness which corresponds to a game where both players can loose. Paraconsistent logic and paracomplete logic appear therefore like husband and wife. The insidious affinity between paraconsistency and paracompletness is expressed in North-American short and imaginative way of speaking by the expression *Gaps and Gluts*.

There are also logics with gaps and gluts. Leaving aside comic strip terminology and using again a neo-classical "para" terminology, such logics, which are both paraconsistent and paracomplete, are called *paranormal logics* or sometimes non-alethic logics (Miró Quesada's terminology). Among paranormal logics, we find De Morgan's logics (see [13], [60]). Paranormal logics may have interesting applications to intensional logic or artificial intelligence. (on paranormal logics see e.g. [15], [61]).

Paraconsistency is an open door to the promising fields of paracomplete and paranormal logics. Therefore, even if in the future it is paracomplete and paranormal logics that will dominate the logical world, paraconsistency will play an essential role in this domination.

3.5 Non monotonic logic, alfabar logic and substructural logics

After rejecting the principles of excluded middle and contradiction, one can wonder what else can be dropped, if we are not already in presence of a naked meaningless conception of logic. But these last years the logicians have gone farther than post-modern artists, they have pursued the deconstruction up to the structural level and study logics called *substructural logics*. The relations between paraconsistent logic and substructural logics may be of various kinds. Paraconsistency can appear as an alternative, replacing the rejection of structural principles, with the rejection of logical principles, escaping the substructural world, or as a complement, crowding the substructural world with contradictions.

Paraconsistent logic can be seen as an alternative, for example, to nonmonotonic logic. Non-monotonists reject monotony because they think that there are experiences (most of the time involving birds) which show that monotony is wrong and in particular leads to some contradictions. But one who thinks the paraconsistent way would reject the principle of non contradiction and not monotony. The strategy of the paraconsistentist is more imaginative, he accepts to see penguins flying in the sky of Hawai's beaches and pink floyds surfing on Antarctica's permafrost. It seems to us that the future shall give the preference to paraconsistent logic taking in account the progress of genitical biology which already produces chicken without feathers, and in the future we may have flying pigs. In such an absurd world, it will make no sense to reason by default, because everything could be true by default.

Paraconsistent logic can also be seen as a complement, for example, to alfabar logics. These are logics in which the law of autodeductibility, $a \vdash a$, is not in general valid (cf. [65]). One can define a paraconsistent negation over such substructural logics. In such logics it may happen that $a, \neg a \nvdash a$. Mixing paraconsistency with alfabaricy may be a good solution to get *neutral paraconsistent logics*, paraconsistent logics in which from a contradiction it is impossible to deduce anything, in exact contrast to classical logic in which a contradiction produces an deductive explosion.

3.6 Non Frego-Aristotelean logic

Consider that the planet LOGOS of the logicians is a rectangle with a length of 100 km and that on the left extermity of the planet (right from the point of view of some Extra-Logosians) there is a big group of serious guys whose god is classical logic they venerate six days a week and that on the right extremity there is a not less numerous crowd of much more agitated people, some of them running in circle trying to catch their own tail, some others trying to jump outside of LOGOS in the middle of nothing. Paraconsistentists seem rather close to the right board of the planet LOGOS. But even if the paraconsistentist is the crazy guy, at the extreme right of the LOGOS planet, trying to jump to another planet, nonetheless he is still on the same logical plane than the classical logician. Both are grounded on some similar Fregean and Aristotelean onto-logical dogmas.

These dogmas are mutliple and form a conception of logic which is still predominant nowadays but seems to be ready to explode. For example one central feature of Frego-Aristotelean logic, its "formal" aspect, is now seriously attacked. It is challenged by people putting meaning into logic but also by people putting meaning onto logic, showing that it is possible to rigorously reason with diagrams such as Venn's diagrams (see [86]). For sure the true revolution of the third millenium logic will be rather connected to a visual logic than to a logic that one gets from classical logic by toying with standard formalizations, dropping one axiom, slightly modifying one other, deleting the left structural rule of right contraction, adding "truth" values, etc, giving birth to monstrous creatures with short life expectancy.

If paraconsistent logic wants to survive the 20th century it should already flirt with non Frego-Aristotelean logics. An interesting easy project, for example, will be to show how to reason with paraconsistent Venn's diagrams. Maybe such paraconsistent diagrams can bring new ideas into the visual anlysis of logic in such a way that paraconsistent logic will not be in the future just an accessory tool but part of the kernel of the new sphere of rationality.

4 Applications of paraconsistent logic

A logic can grow like a majestic tree, dominating the logical florest by its beauty and grandiosity, but if such a tree produces no fruits, it can turn into a monstrous cadaver of black wood soon to disappear.

Like for many non classical logics, the future of paraconsitent logic will in great part depends on its possible applications. Let us therefore examine if paraconsistent logic is a fruitful tree, or a sterile plant that must be removed from the garden of science.

4.1 Applications to human sciences

Maybe human being is the only natural phenomenon which is contradictory, producing contradictions and seeing them everywhere. Obviously human being is full of contradictions. The question is to know if they must be bannished and rejected considering them as errors and source of confusions, or if they must be taken as "natural", an essential part of human nature we must deal with.

Given a human being like John Smith, with contradictory desires and wills, can we think that it is his normal state, that John Smith behaves in a paraconsistent way and that paraconsistent logic is the adequate tool to describe his behaviour? Or must we think that these contradictions are a kind of disease that should be eliminated, for John Smith recovering his health, following again the pattern of classical logic?

Are contradictions irrational? Is paraconsistent logic the logic of parapsychology, lacanian inconscious, sokalian impostures, cyber yuppies? Is paraconsitent logic just a curiosity which fits well the spirit of the post-modern bric-a-brac of new age rationalism? Or is paraconsistent logic the logic of political correctness, of minorities, pluralities, the logic of oecumenical concilianility of the Aquarian era?

It is difficult to answer all these questions. For the present time, there is on the one hand an odd mathematical tool and on the other hand apparently contradictoy phenomena, described with languages which have structures very different to mathematics.

We must investigate, little by little, applications of paraconsistent logic and mathematical modelizations of human phenomena to see if we find a convergent point. One can start by working with more concrete cases, such as for example the case of lie dectectors and contradictory witnesses [27].

4.2 Applications to natural sciences

Are there contradictions in the nature ? Let us first note that someone who believes in real contradictions does not necessarily believe in natural contradictions, since for example linguistic contradictions may be considered as real but not part of the nature. It all depends of course of how we define and conceive nature. But in general linguistic is considered as a human science by opposition to natural sciences such as physics and biology.

Contradictions can appear in many different ways within physics. Some people think that the physical world is bound by the interelationship of antagonistic forces. Anyway the underlying standard logic of physics is classical logic. This does not mean that another logic based on contradictions could not lead to an equivalent theory or a better one, but until now such kind of possibility has not been developed in a way as systematic and general as the standard framework. Even in the case of quantum physics, where contradictions appear in a phenomenon such as the duality wave/particle.

The people of the Copenhague's school have developed an intepretation of this duality based on the notion of complementarity. Such an interpretation can be seen as having a touch of paraconsistency. But Copenhague's interpretation was presented rather in a philosophical informal way by Bohr and Heisenberg. Paraconsistency could be used as a possible formalization of it, as alternative to other previous formalizations such as the many-valued logic of P.Février [59], a logician who took seriously the idea of complementarity. People like N.C.A. da Costa and D.Krause are presently working on a paraconsistent approach to complementarity [55]. Another Costa's idea involving paraconsistent logic into the formalization of physics, is *multideductive logic*, a way to unify contradictory theories, worked out in details by E.G. de Souza [88]. In the future, if nobody succeeds to unify micro and macro physics into a "classical" theory, maybe paraconsistent logic will be an essential tool for the foundations of physics.

Biological phenomena can even more easily be interpreted as contradictory phenomena . Hegel used to take the growth of a plant as a typical example of contradictory natural phenomenon. The question it know how this contradiction-oriented approach can be systematized and fromalized. Until now, no serious mathematical system based on contradictions has been proposed for biology. But it is also true that in general, applications of logic to biology are quite rare despite of the interest of Aristotle and Tarski for this science. Maybe it is because classical logic does not fit at all biological phenomena. Maybe then in the future, a logic like paraconsistent logic will be useful to turn biology logical.

4.3 Applications to formal sciences

By formal sciences, we understand here logic, mathematics and any other general abstract nonsense.

When, a century ago, Cantor created a paradise full of alephes, and other sensual creatures that seem to be only product of a endless imagination, he was arguing against the theologians, that mathematics is free, in the sense that the mathematician can entertain himself with any kind of creatures, unless there are contradictory. Contradictions are the end of the game. Contradictions are like snakes in Cantor's paradise. If you meet a snake, he will bite you and you will have to go out of the paradise of alephes, awaking in everyday world, full of politicians and pollution.

The mathematician is therefore in some sense totally free, he can create anything even if these things have no applications or are not in any sense representations of reality. But in another sense he is not so free : he is severely bound by the principle of non contradiction. He cannot, like the poet, live in a world of flying rabbits, happy alephes and true liars.

Maybe the paraconsistentist wants to turn mathematics more poetical, allowing Russellian sets and other funny creatures in Cantor's paradise, such as friendly snakes.

However it seems that paraconsistent logic does not broaden significantly enough the mathematical horizon. If we replace consistency with non triviality, we get a mathematical world which is much the same as the classical world. It is not clear to which point this paradigmatical change permits to introduce new mathematical concepts of interest. New creaures, like a Russellian set, are maybe only funny ugly ornaments added to the beautiful classical architecture of mathematics, as the gargoyles added to the majestic architecture of *Notre Dame*.

But maybe these new creatures authorized in the mathematical paradise by paraconsistent logic will play a central role in the future scene of mathematics, maybe paraconsistent logic will give, at least, definitive citizenship to infinitesimals in the mathematical country (cf. [72]). Or maybe paraconsistent logic will save us from the tricephalous CGC-monster (CGC for Cantor-Gödel-Church) by providing foundations for finite decidable complete mathematics (see [14]). It is too soon to have an opinion, but against the skeptics who see these creatures as disturbing parasites, toys for infantil logicians, we can recall George Birkhoff's remark : "It is well not to forget that many of the most astonishing mathematical developments began as a pure *jeu desprit*."

4.4 Applications to cyber sciences

The expression "cybernetics" is a word from the prehistory of our modern computer times that has started to be used again in the eighties in a shorter design (cf. cyber space). Following Prof. Castigo, from the Santa Boneca Institute, we will use it in its shorter design, as a generic term for information sciences, cognitive science, artificial intelligence, computational intelligence, etc, all these sciences converging towards the birth of a super robot, a cyber robot.

For sure cyber science will rule the future in a way that we can imagine only superficially. Let us see if paraconsistent logic will play a significant role in this future or if we shall be regulated by classical bivalent robots not even able to laugh at a contradiction.

The Prof. Tsujiski just informed us about a recent experience that has been performed in the Butatin Studio of the Center for Avanced Study of the University of São Paulo, under the guidance of Prof. Abe [1]. A paraconsistent female robot called Sophia and her brother Anti-Sophia, have survived a confrontation with tigers, snakes and monkeys using the paraconsistent logic "pétale" which rules the electronic circuits of their neural networks (But Prof. Tsujiski said nothing to us about the ability of these paraconsistent robots to survive the stress of everyday life in Wall Street).

In our opinion this experience, as well as other similar experiences (cf. [56]), show very well the future of paraconsistent logic: We don't know if paraconsistent logic is just a crazy dream of our present times that will soon disappear or the new logic of a future wonderful more colourful world, but fore sure tomorrow, even in a not so wonderful world, there will be a cohort of intelligent paraconsistent robots ruling our world, maybe with many fuzzy brother robots, and many non monotonic sister robots. However we cannot predict if their big brother will be a pure paraconsistent robot, or a substructural paraconsistent turbo polar robot [75]).

Maybe in the future paraconsistent logic will go out of the logico-mathematical scene, manifesting itself in an important way, only in the cyber engineer scene.

5 Paraconsistent logic and the third millenium

Some paraconsistent new agers like to say, parodying Malraux, that the third millenium will be paraconsistent or will not be. We don't know exactly what they mean. For sure, if we succeed to escape big-bug annihilation, it will not be via paraconsistent logic. The big-bug problem is probably full of contradictions, but with January 1st 2000 approaching at a speed of 2160 seconds per hour it is very unprobable that paraconsistent logic can help in our salvation, since it has not even yet been applied to bug problems. So if we enter the third millenium this will be thanks to old black and white binary classical logic. But of course this does not mean that classical logic will be able to guide us through the whole third millenium. Maybe paraconsistent logic will take the relay and help us to cope with the next big-bang.

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Jean-Yves Béziau
Universidade Federal Fluminense
Instituto de Matematica
Departamento de Analise
Niteroi, Rio de Janeiro, Brazil
jybeziau@hotmail.com

Contents 1. Logic of the future or logic without future ? 2. Paraconsistent problems 2.1. The problem of definition of paraconsistent logic 2.2. Philosophical problems 2.3. Mathematical problems 3. Relations between paraconsistent logic and other logics 3.1. Fuzzy logic and many-valued logic 3.2. Relevant logic 3.3. Modal and intensional logics 3.4. Intuitionistic logic, paracomplete logics and paranormal logics 3.5. Non monotonic logic, alfabar logic, substructural logics 3.6. Non Fregoâ€¦ CONTINUE READING. A paraconsistent logic is a logical system that attempts to deal with contradictions in a discriminating way. Alternatively, paraconsistent logic is the subfield of logic that is concerned with studying and developing paraconsistent (or "inconsistency-tolerant") systems of logic.Â However, if we know that either P or A is true, and also that P is not true (that Â¬P is true) we can conclude that A, which could be anything, is true. Thus if a theory contains a single inconsistency, it is trivialâ€"that is, it has every sentence as a theorem. The characteristic or defining feature of a paraconsistent logic is that it rejects the principle of explosion. As a result, paraconsistent logics, unlike classical and other logics, can be used to formalize inconsistent but non-trivial theories.