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Transmission Line Transformers: Theory, Design and Applications — *Part 1*

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Here is a thorough treatment of the operation and practical design of low-loss, wide bandwidth transmission line transformers

The concept of transmission line transformers (TLTs, also known as equal delay transformers) has been a distinct element of RF circuit design since at least 1944, when Guanella disclosed an impedance transformer of novel design which consisted of a pair of interconnected transmission lines [1, 2]. TLTs have been found to possess far wider bandwidth and much greater transmission efficiencies by arranging the windings of the TLT to have uniform transmission line properties. In general, these devices are widely used for matching networks for antennas and amplifiers in the HF and VHF bands [3], and their low losses (a fraction of a dB) make them especially useful in high power circuits [4].

Typical structures for TLTs consist of parallel wires [5], coaxial cable, or bifilar twisted wire pairs [6, 7], with the latter being popular as the characteristic impedance can easily be determined by the wire diameter, the insulation thickness, and to some extent, the twisting pitch [7, 8]. In the case of coaxial cable transmission line with the correct characteristic impedance for the TLT, the theoretical high frequency bandwidth limit is reached when the cable length comes into the order of a half wavelength (λ/2), with the overall achievable bandwidth being about a decade [6].

By introducing magnetic materials such as powdered iron or ferrite [9, 10] to the TLT, both the low frequency limit and the high frequency limit are improved [6], and when low loss, high permeability ferrites are used along with good quality semi-rigid coaxial cable, bandwidths of four decades or more are achievable [11].

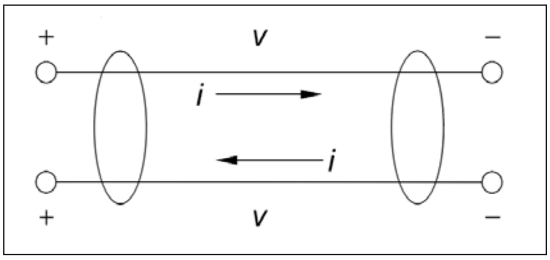

Figure 1 · Transmission line in transverse (TEM) mode.

What's All This TEM Stuff, Anyhow?

Transmission line transformers operate by transmitting energy by way of the transverse (or TEM, meaning Transverse Electro-Magnetic [12] or Transverse Electric and Magnetic [13]) transmission line mode, rather than by the more familiar coupling of magnetic flux as with a conventional transformer [4]. Figure 1 illustrates this concept in generalized form, where the two lines represent the two conductors of a transmission line, regardless of whether it is made of parallel wires, twisted wires, coaxial cable, or any other means. Here, the currents in the two conductors are equal in magnitude and opposite in phase, while the voltages along the two conductors are equal in magnitude [14, 15]. In the TLT, the windings serve to eliminate, or at least substantially reduce, common-mode currents from the input to the output [6].

The terms transverse electric (TE) and transverse magnetic (TM) refer to conditions in which the electric field or magnetic field, respectively, of a propagating wave is parallel to a boundary plane, in this case being the surface of the conductors of the transmission line, while at the same time the accompanying

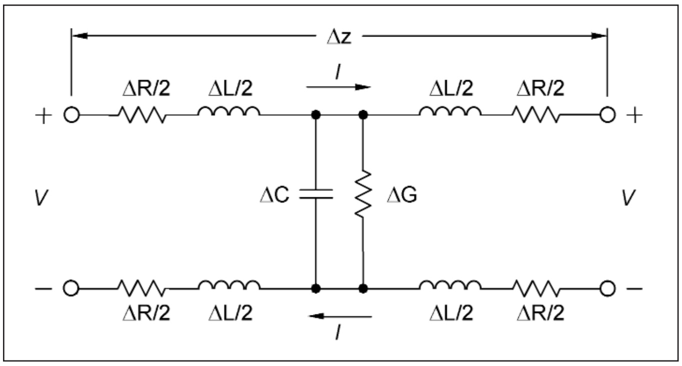

Figure 2 · Incremental model of TEM transmission line.

magnetic or electric fields, respectively, still have some longitudinal (or axial) components [15]. The term transverse electromagnetic (TEM) refers to a condition in which both the electric and magnetic fields are parallel to the boundary plane [14], and there are no longitudial components of either field.

Generalized TEM Transmission Lines

Let's examine the parameters that are common to all forms of TEM mode transmission line, and we'll later do a detailed study of coaxial cable as it is the easiest to comprehend since all of the electric and magnetic fields are contained between the conductors when in TEM mode. For all types of TEM mode transmission lines (coaxial, parallel wire, twisted wire, waveguide, etc.) the equations have the same basic form. Referring to Figure 2, if R, L, G and C are the total series resistance, series inductance, shunt conductance, and shunt capacitance per unit length z, then the transmission line may be expressed as [14]:

$$\frac{\partial V}{\partial z} = -(R + j\omega L)I \tag{1}$$

$$\frac{\partial I}{\partial z} = -(G + j\omega C)V \tag{2}$$

which are derived from Maxwell's theorems and equations [14, 15, 16]. We'll dispense with the usual two or more chapters of differential calculus that normally bring us to this point.

From (1) and (2) we can derive the characteristic impedance Z_o of the transmission line as related to the primary constants R, L, G and C by [14]:

$$Z_0 = \frac{\partial V}{\partial I} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{3}$$

and the complex propagation constant γ, which can be approximated as [14]:

$$\begin{aligned}\gamma &= \alpha + j\beta = \\ &= \sqrt{(R + j\omega L)(G + j\omega C)}\end{aligned} \tag{4}$$

where α is called the attenuation constant and β is called the phase constant. The primary constants R, L, G and C are directly related to the physical properties of the materials used in the transmission line and remain unaffected by the application of the transmission line.

For low-loss transmission line such as good quality coaxial cable [14]:

$$R \ll \omega L \tag{5}$$

$$G \ll \omega C \tag{6}$$

allowing the characteristic impedance Z_o to be approximated as [14]:

$$Z_0 \approx \sqrt{\frac{L}{C}} \tag{7}$$

and the complex propagation constant γ to be approximated as [14]:

$$\gamma \approx j\omega\sqrt{LC} \tag{8}$$

The reciprocal of the square root of the product of L and C provides us with the velocity of propagation or phase velocity [14,17]:

$$\upsilon_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} \tag{9}$$

where μ is the permeability of the transmission line medium in henries/meter (H/m) and ε is the permitivity of the transmission line medium in farads/meter (F/m) [14].

For purposes of circuit analysis, the TEM transmission line of Figure 1 can be described as a 2-port ABCD matrix, as shown in Figure 3 [18] and analytically as:

$$\begin{bmatrix} V_{\text{in}} \\ I_{\text{in}} \end{bmatrix} = \begin{bmatrix} \cosh\gamma\, I & Z_0 \sinh\gamma\, I \\ \dfrac{\sinh\gamma\, I}{Z_0} & \cosh\gamma\, I \end{bmatrix} \times \begin{bmatrix} V_{\text{out}} \\ \mathrm{I}_{\text{out}} \end{bmatrix} \tag{10}$$

$$V_{\text{in}} = V_{\text{out}} \sinh\gamma\, I + I_{\text{out}} \cosh\gamma\, I \tag{11}$$

$$I_{\text{in}} = \frac{V_{\text{out}}}{Z_0} \sinh\gamma\, I + I_{\text{out}} \cosh\gamma\, I \tag{12}$$

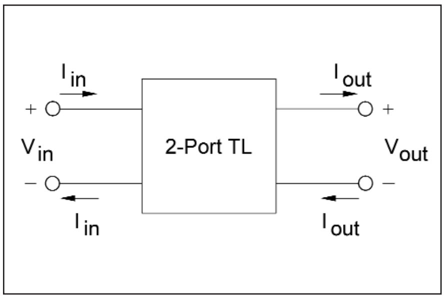

Figure 3 · TEM transmission line as 2-port ABCD element.

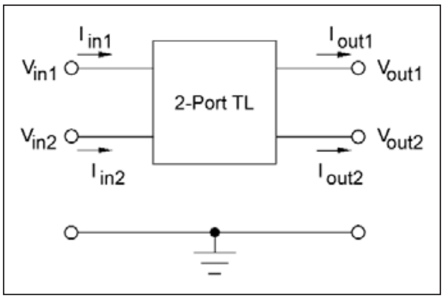

Figure 4 · 2-port TEM transmission line as 4-port ABCD element.

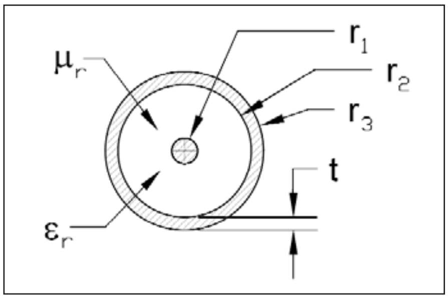

Figure 5 · Coaxial transmission line.

The 2-port transmission line of Figure 1 may also be described as a 4-port ABCD matrix [19] where each terminal of the transmission line is paired with ground, as shown in Figure 4, and described analytically as:

$$I_{\mathrm{in2}} = -I_{\mathrm{in1}} = I_{\mathrm{in}} \tag{13}$$

$$I_{\mathrm{out2}} = -I_{\mathrm{out1}} = I_{\mathrm{out}} \tag{14}$$

$$V_{\mathrm{in1}} = V_{\mathrm{out1}} \cosh\gamma\, I + Z_0 I_{\mathrm{out}} \sinh\gamma\, I \tag{15}$$

$$V_{\mathrm{in2}} = V_{\mathrm{out2}} \cosh\gamma\, I + Z_0 I_{\mathrm{out}} \sinh\gamma\, I \tag{16}$$

$$\begin{aligned} I_{\mathrm{in}} &= V_{\mathrm{out1}} \sinh\gamma\, I - V_{\mathrm{out2}} \sinh\gamma\, I + I_{\mathrm{out}} \cosh\gamma\, I = \\ &= \left(V_{\mathrm{out1}} - V_{\mathrm{out2}}\right)\sinh\gamma\, I + I_{\mathrm{out}} \cosh\gamma\, I \end{aligned} \tag{17}$$

The second term of Equations (15) and (16) represent the voltage along the length of the transmission line, and it can be readily seen that this voltage is equal in magnitude and phase for both conductors. As we shall see later, this voltage term is commonly used in the design of TLTs as it is more convenient than using the end voltages, and it does not detract from the voltage and current characteristics of the transmission line.

Coaxial Cable

Let's now turn to the specific case of coaxial cable, using the illustration of Figure 5 as a guide. Here, the inner conductor has a radius of r_1 and the outer conductor has an inner radius of r_2, an outer radius of r_3, and a thickness of t. The space between the inner and outer conductors is filled with an insulating dielectric material such as PTFE that has a relative permitivity (or dielectric constant) ε_r and a relative permeability μ_r. If the conductors are lossless, as per our earlier approximation, the unit shunt capacitance C is [17]:

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{55.6\,\varepsilon_r}{\ln\left(\frac{r_2}{r_1}\right)}\ \mathrm{pF/m} \tag{18}$$

and the unit series inductance L is [17]:

$$\begin{aligned} L &= \frac{\mu}{2\pi}\ln\left(\frac{r_2}{r_1}\right) = \\ &= 0.2\,\mu_r \ln\left(\frac{r_2}{r_1}\right)\mu\mathrm{H/m} \end{aligned} \tag{19}$$

The unit series resistance R is related to the conductivity of the conductors and is frequency dependent by way of a phenomenon known as skin effect. We begin by first defining a quantity known as the $1/e$ depth of penetration [15,17,20]:

$$\delta \approx \sqrt{\frac{2}{\omega\mu_v\mu_r\sigma}} \tag{20}$$

where μ_v is the permeability of free space ($4\pi \times 10^7$ H/m) and $\alpha\ \nu\ \delta\ \sigma$ is the conductivity of the material. The current in a conductor will always concentrate on the surface that is nearest the wave that creates the current [17], and in the case of coaxial cable this is the electromagnetic field that exists between the inner conductor and the inner surface of the outer conductor. At high frequencies the skin effect causes the current to flow only on the outer surface of the inner conductor and the inner surface of the outer conductor [17], and this condition persists as long as the thickness t of the outer conductor (see Figure 5) is appreciably greater than the depth of penetration.

From (20) we can now define the surface resistance of the conducting material [14]:

$$R_s = \frac{1}{\sigma\delta}\,\Omega \tag{21}$$

The unit shunt conductance is related to the resistivity of the dielectric insulating material, due to a phenomenon known as dielectric hysteresis, which is analagous to the magnetic hysteresis in magnetic materials [15]. It is convenient to describe the total losses of the transmission line as the equivalent conductivity [15]:

$$\sigma' = \sigma + \omega\varepsilon'' \tag{22}$$

from which we can derive the loss tangent of the transmission line, which is [15]:

$$Tan\ \delta = \frac{\sigma'}{\omega\varepsilon'} \tag{23}$$

where ε´ and ε´´ are often referred to as dielectric dispersion formulas [15,21], which are very rigorous and very much beyond the scope of this discussion. It is sufficient for our purposes to use the loss tangent data provided by the manufacturer of the cable we are using and from that derive the surface resistance R_s.

Going back to (9), we can define a quantity which is known as the velocity factor of the cable:

$$VF = \frac{\upsilon_p}{c} = \frac{\sqrt{\mu_v\,\varepsilon_v}}{\sqrt{\mu\,\varepsilon}} = \\ = \frac{\sqrt{\mu_V\varepsilon_V}}{\mu_r\,\mu_v\,\varepsilon_r\,\varepsilon_v} = \frac{1}{\sqrt{\mu_r\,\varepsilon_r}} \tag{24}$$

where c is the speed of light and ε_v is the permitiviity of free space (8.854 × 10^{-12} F/m). In general the relative permeability of most, if not all insulating materials is close to unity, so (24) can be comfortably approximated as:

$$VF \approx \frac{1}{\sqrt{\varepsilon_r}} \tag{25}$$

Coaxial cable is a good example of TEM transmission line as the skin effect causes the current of the outer conductor to be concentrated on the inner surface. The magnetic fields generated by the equal and opposite currents of the concentric inner conductor and the inner surface of the outer conductor cancel outside the outer conductor in both the near and far fields, leaving no net magnetic field outside of the outer conductor that would couple to nearby objects, such as the magnetic core of a TLT, which would cause additional losses beyond those of the cable itself [17,22].

Similar equations may be developed for parallel wire transmission line [14,15,16] as well as twisted wire transmission line [8], the latter of which is an important element in the design of wideband transformers for RF applications [23].

Low Frequency Bandwidth Limit

The low frequency bandwidth limit of a TLT made with coaxial cable is determined by way of the magnetizing inductance of the outer surface of the outer conductor, which results in the low frequency model illustrated in Figure 6 [3,24]. Here, the transmission line proper is represented by the ideal 1:1 transformer. The resistance R_0 represents the losses of the transmission line, and the inductance L_{ac} represents the magnetizing inductance of the outer surface of the outer conductor. Note that there is no parallel inductance for the inner conductor, which is due to the fact that the series inductances of the inner conductor and the inner surface of the outer conductor are part of the transmission line proper [24].

An approximation to the magnetizing inductance can be made by considering the outer surface of the coaxial cable to be the same as that of a straight wire (or linear conductor) which, at higher frequencies where the skin effect would cause the current to be concentrated on the outer surface of the outer conductor, would have the self-inductance of [25]:

$$L_{ac} = 2l\left[\ln\left(\frac{2l}{r_3}\right) - 1\right]\text{nH} \tag{26}$$

where l is the length of the coaxial cable in cm and r_3 is the radius of the outer surface of the outer conductor in cm, as shown in Figure 5. As stated, this inductance is for a straight conductor, therefore it will generally increase when the coaxial cable is formed into various shapes such as a

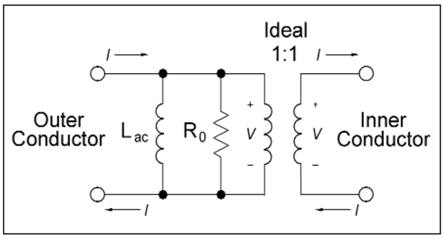

Figure 6 · Low frequency model of 1:1 transmission line transformer using coaxial cable.

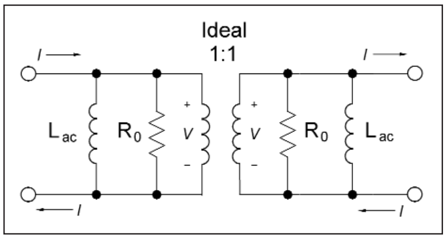

Figure 7 · Low frequency model of 1:1 transmission line transformer using twisted or parallel wires.

helix, making the inductance L_{ac} of (26) a lower limit in the design process.

A similar low frequency model for transmission line transformers using twisted or parallel wires is shown in Figure 7 [26]. Here, the model is symmetrical as both conductors are exposed to the magnetic material and therefore both contribute to the losses and low frequency characteristics of the transformer. In cases where the transmission line is greater than an eighth of a wavelength, it may be more appropriate to replace the magnetization inductors with individual sections of transmission line [26].

This is a matter that is understood by way of sufficient practical experience more than anything else. As a general rule, the length of the transmission line is kept to no more than an eighth of a wavelength ($\lambda/8$) at the highest frequency for the application.

For best performance, the characteristic impedance of the transmission lines used in the TLT should be equal to the geometric mean of the input and output impedances [26, 27, 28]:

$$Z_{TL} = \sqrt{Z_{\text{in}} \times Z_{\text{out}}} \tag{27}$$

although nonoptimal transmission line characteristic impedances may be used provided that the increased losses, degraded return loss, and reduced bandwidth are acceptable, which is sometimes a necessary tradeoff when using commercial coaxial transmission line.

From (26) and (27) we can now estimate the low frequency bandwidth limit of a TLT using coaxial cable by:

$$f_L = \frac{Z_{TL}}{2\pi L_{ac}} \tag{28}$$

Referring back to the low frequency models of Figures 6 and 7, take note of the fact that the voltage for the low frequency model is described along the length of the conductors rather than across the end terminals, which is represented by the second term of equations (15) and (16). This voltage orientation is commonly used in the design of TLTs as it is more convenient and it does not detract from the voltage and current relationships of the transmission line.

Next Month

This article will conclude next month with the effects of magnetic materials and methods for the realization of practical TLTs with the desired impedance transformations.

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